1 Billiards

Click the Graphics button in the Billiards text window to open the Billiards graphics window (as shown in Figure II, in the last section).

Aim: to demonstrate ELASTIC collisions between two macroscopic particles (billiard balls), which obey the non-relativistic equations for the conservation of KINETIC ENERGY and MOMENTUM. You can calculate the mass of the incident particle knowing the recoil angles of the balls and the speed of one of them.

Click the 

button in the graphics window. This causes a white ball of unknown mass, m_1, to move on a collision course towards a stationary red ball, initially positioned in the centre of the viewport with mass, m_2 = 3 Kg. After the collision the white ball is deflected through an angle *q* with respect to its initial direction of motion. Likewise the red ball is deflected through an angle *f* with respect to this same direction (see Figure 1.1).

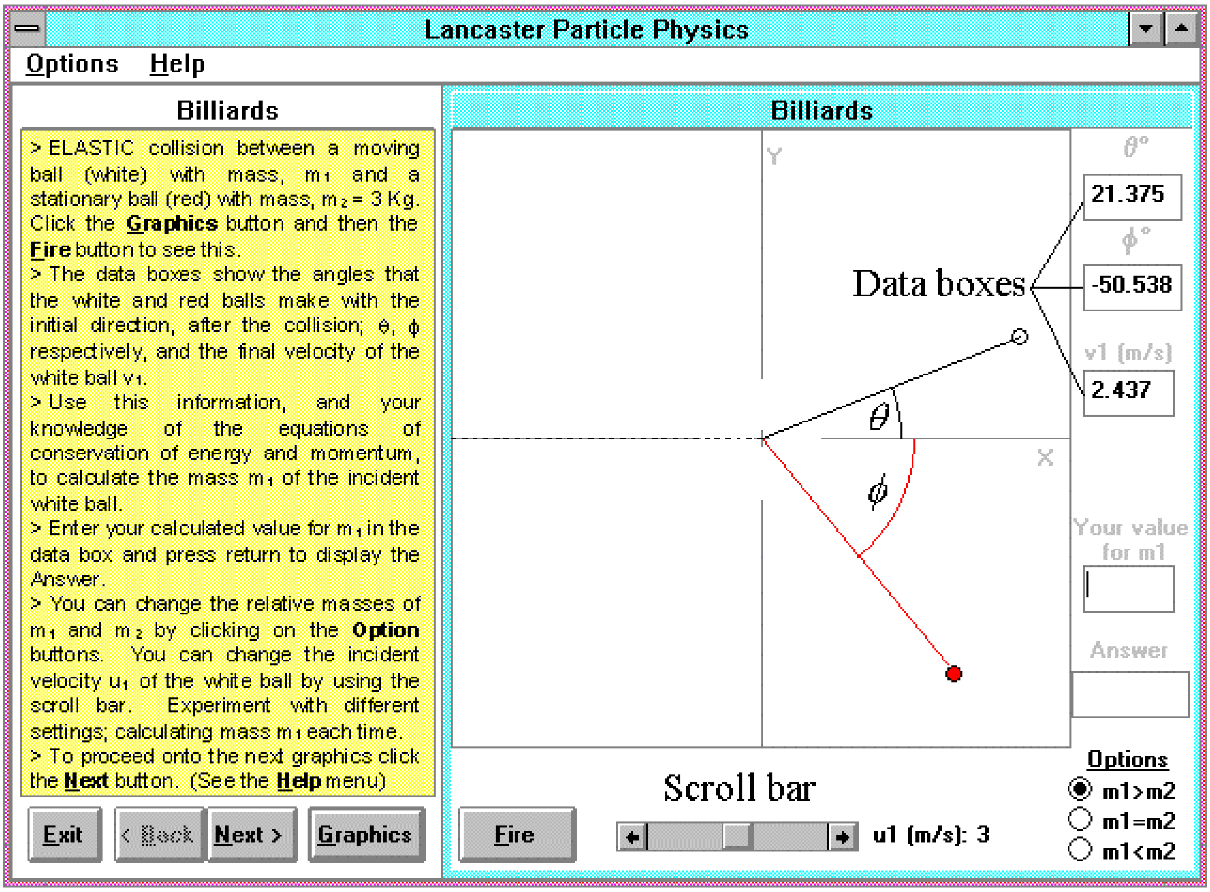

Figure 1.1

Note: in all the options, the Fire button is disabled (appearing dimmed) from the moment it is clicked until the end of the graphics simulation. This prevents accidental firing if the button is clicked more than once.

The final velocity of the white ball, v_1 , and the two angles θ and ϕ are displayed in the data boxes, at the end of each firing sequence - when the simulation ends. The initial velocity of the white ball is known from the scroll bar data box (u_1) - this has a default setting of 3 ms^{-1} upon opening the graphics window. The scroll bar allows the initial velocity of the white ball to be changed and set, in the range from 1 to 5 ms^{-1}, prior to firing. To change the scroll bar setting click on the arrow keys or drag the central scroll bar button (thumb) using the mouse pointer (see Figure 1.1).

When the collision is complete you should note all the information in the data boxes. Your task is to calculate the unknown mass, m_1 of the white ball. We are neglecting non-linear effects such as friction, air drag and ball spin; therefore, because the collision is elastic, the total kinetic energy is conserved. Conservation of linear momentum in the direction of the incident ball (*i.e.* horizontally), gives

$$m_1 u_1 = m_1 v_1 \cos\theta + m_2 v_2 \cos\phi \qquad (1.1)$$

Conservation of kinetic energy gives

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \qquad (1.2)$$

We can solve these two simultaneous equations to determine the unknown mass, m_1. Try this for yourself - the correct expression for m_1 is given at the end of this section. (Note: an alternative and simpler form of equation 1.1 can be written by resolving the momentum in the vertical direction. Solving for m_1 should then give you a similar expression.)

To see if your value for m_1 is correct, enter it into the data box containing the flashing cursor and press return. This will cause the correct value to be displayed (in red) in the Answer box, as shown in Figure 1.2. (The data box is enabled at the end of each firing sequence) Note: if you enter an invalid entry, such as a character instead of a numerical value, then an *ERROR!* dialogue box will appear. Alternatively, if your entered value is over 50% larger or smaller than the correct answer, then a *TRY AGAIN!* dialogue box will appear, telling you to recalculate your value. Click the OK button to clear both the dialogue boxes. This will also cause the data box to clear, after which you can enter your new value. (Hence, the answer will only be displayed, if you correctly enter your value for m_1 and it is within ± 50% of the actual value)

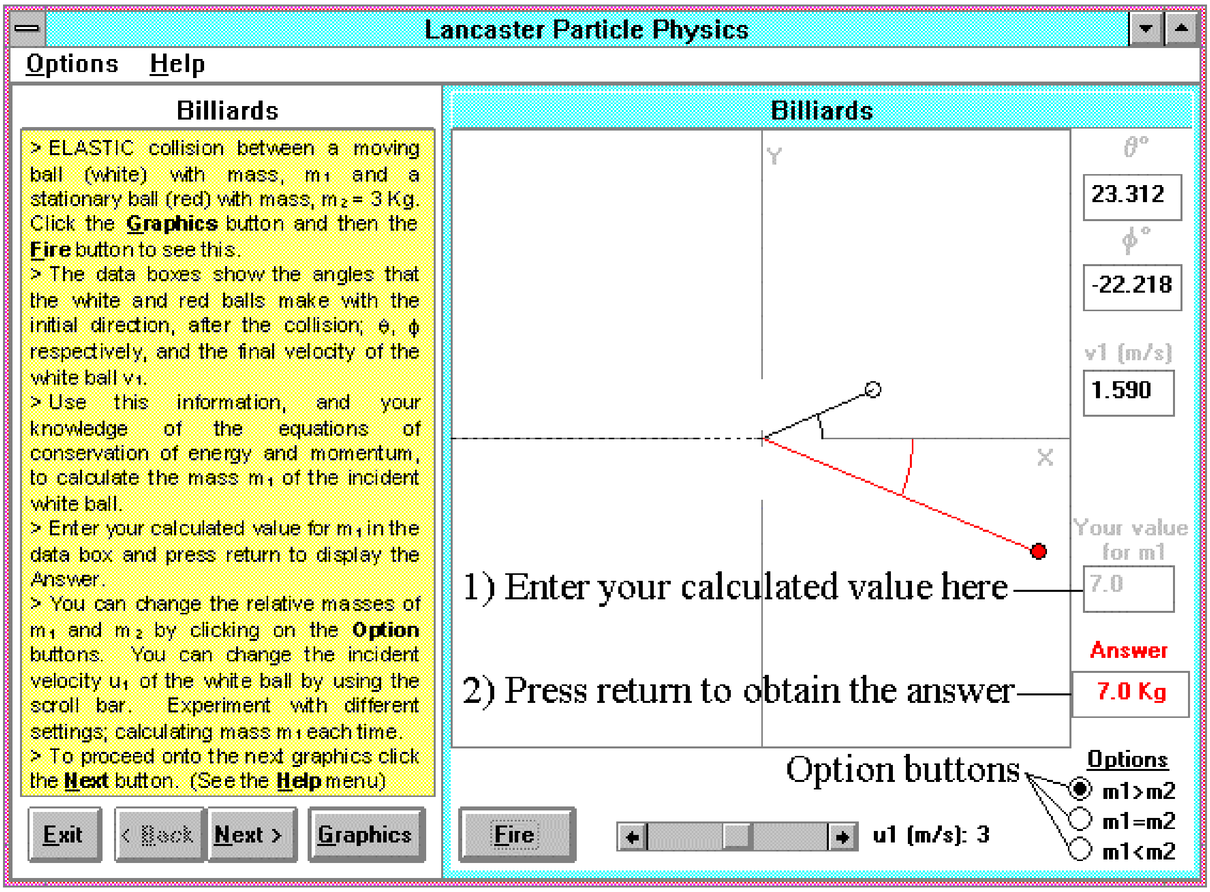

Figure 1.2 To display answer, enter your value into the data box and press return.

You can change the relative masses of m_1 and m_2 by clicking with your mouse pointer, on the round Option buttons located in the graphics window. When selected a dot appears inside the option button (see Figure 1.2). When changing options only the mass m_1 varies; m_2 always stays equal to 3 Kg. The three options are:

- $m_1 > m_2$ m_1 has a variable mass > 3 Kg and ≤ 30 Kg.
- $m_1 = m_2$ m_1 has a fixed mass of 3 Kg.
- $m_1 < m_2$ m_1 has a variable mass ≥ 0.1 Kg and < 3 Kg.

(Upon opening the graphics window the default setting is $m_1 > m_2$)

Note: m_1 is different on each new firing sequence (except when $m_1 = m_2$); its value will lie in the range specified by the option buttons.

For $m_1 = m_2$ the included angle $(\theta + \phi)$ between the two balls, after the collision, is equal to 90° *i.e.* the final velocities of the two balls are perpendicular; a phenomenon familiar to billiard and pool players (in real life the effects of friction makes this angle somewhat less than 90°).

1.1 Summary of calculation

Step 1: After the firing sequence has ended, note down the scroll bar setting and all the information in the data boxes.

Step 2: Write out equation 1.1.

Step 3: Write out equation 1.2.

Step 4: Solve the simultaneous equations to give an expression for m_1 in terms of m_2, u_1, v_1, θ and ϕ. (See below)

Step 5: Substitute values to calculate m_1.

Solution:

$$m_1 = \frac{m_2 \cos^2 \phi \left(u_1^2 - v_1^2\right)}{\left(u_1 - v_1 \cos\theta\right)^2} \quad (1.3)$$

Aside: Relativistic Billiards

When billiard balls (or particles) approach the speed of light, *c*, their properties are affected according to special relativity.

Relative to a stationary observer, the moving particle (speed *v*) behaves as if its mass (m_0) is greater, than when it is as rest, by the Lorentz factor: $g = 1\big/\sqrt{1-b^2}$ where $b = v/c$. So momentum becomes

$$p = g\, m_0\, v \qquad \text{or} \qquad p = g\, m_0\, b c \tag{1.4}$$

The energy of the particle becomes

$$E = g\, m_0\, c^2 \tag{1.5}$$

This energy *E* is called the *total* energy of the particle because it is made up of its rest energy, $m_0 c^2$, and its kinetic energy. We can relate the total energy *E* directly to *p* by combining equations 1.4 and 1.5 to eliminate the particles velocity, to give

$$\boxed{E^2 = m_0^2 c^4 + p^2 c^2}$$

Try this for yourself. Can you also show that, when *b* becomes much less than 1, this reduces to the non-relativistic form: $E = m_0 c^2 + \frac{1}{2} m_0 n^2$.

Time also appears to go slow for the moving particles. A time interval Δt for the particle becomes lengthened to $\gamma\Delta t$, as measured by the stationary observer. This is particularly important when measuring the lifetime of particles which are travelling at relativistic speeds - see section 4 ‘Lifetime’.