4 Lifetime

When you have completed the ‘Motion in a magnetic field’ exercise, click the Next button in the text window to move onto the next section. Or select ‘Lifetime’ from the Options menu. Either way the text window for ‘Lifetime’ will appear.

Aims: to determine 1) the rest mass and 2) lifetime of a neutral particle, the kaon (K^0), by studying its decay products a p^+, p^- pair and its decay length; distance travelled by the K^0 from creation to decay.

Click the Graphics button in the text window to display the graphics window, as shown in Figure 4.1. This shows the same *xy*-view as in the previous exercise. The electron and positron beams travel along the axis of the accelerator, *i.e.* into and out-of the screen; they meet and annihilate in the centre of this view at the origin of the *x*, *y* axes.

Click the Fire button in the graphics window. This causes an annihilation as indicated by the *POW!*. However, this time the electron and positron annihilate to form a K^0 (yellow ball), and a JET of particles (grey wedge): we assume that the JET and the K^0 each has a total energy equal to the Incident beam energy. The K^0 travels in a straight line and leaves no track because it has zero electrical charge. After a period of time, usually short, the K^0 decays into a p^+, p^- pair (green and blue balls) its position of decay indicated by the white ‘ghost’ ball. The charged pion pair are affected by the uniform magnetic field, coming out of the screen, and follow curved trajectories (see Figure 4.1).

The Jet of particles usually consists of pions; their presence here is not important for this exercise and so they can be ignored. Change the Incident beam energy and Magnetic field strength by using the scroll bars, positioned at the bottom of the graphics window. Click Fire for each new setting of the scroll bars, and notice the form of the K^0 decay. There are several points to consider

1) as with the charged particle pairs in the previous exercise the K^0 has no preferred direction. On each new firing sequence its direction of motion is different. Likewise the JET has no preferred direction, however, relative to the kaon it always points in the opposite direction in order to conserve momentum. Notice also that as the beam energy is increased the angular spread of the Jet decreases.

2) Conservation of momentum causes the pions to be created travelling in the same general direction as the K^0 motion, before decay. The pions also follow trajectories with different radii on each new firing sequence, since they have different momenta on each decay.

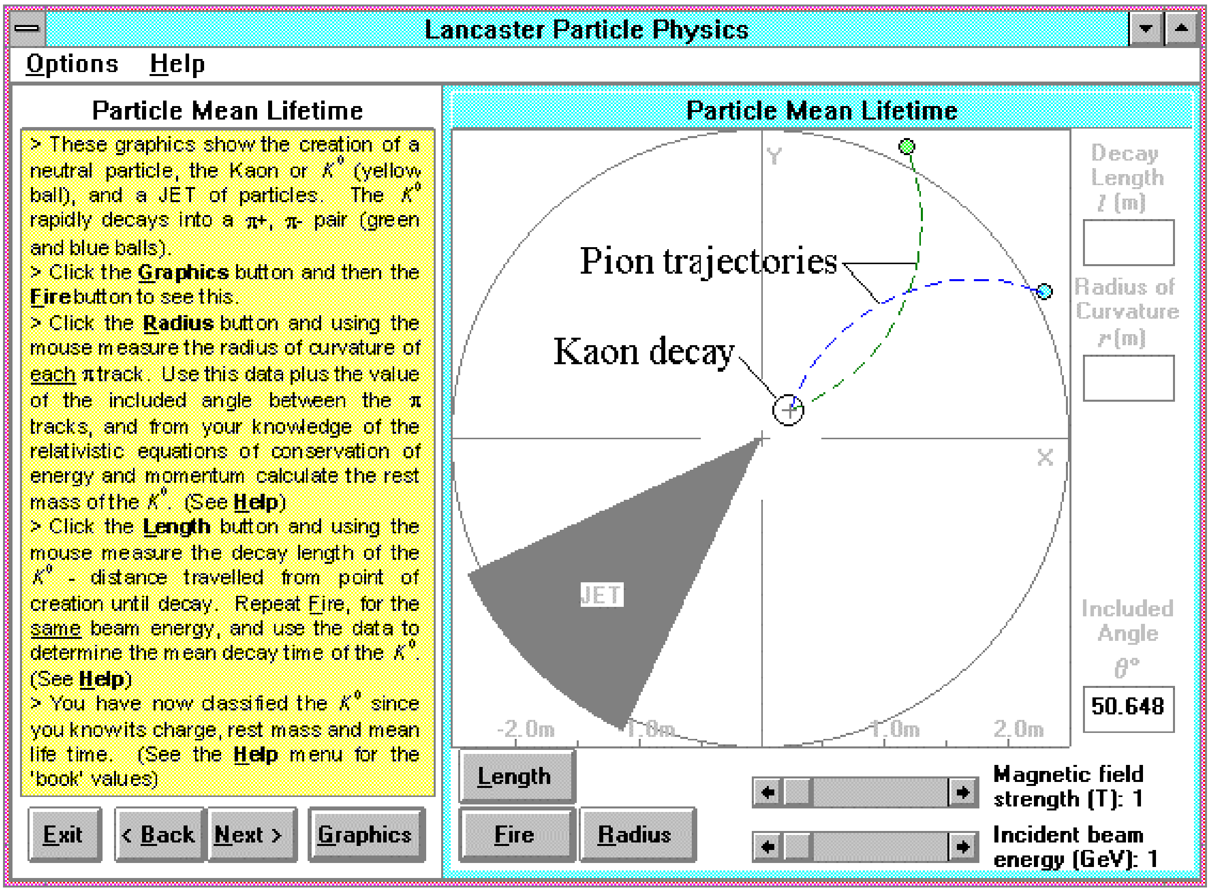


Figure 4.1 'A typical Firing sequence'

Despite changing on each firing sequence, the form of the pion trajectories fall into two main categories. Their tracks either, initially diverge before coming back on themselves and crossing, or they diverge and never cross (see Figure 4.2a, b). This is a consequence of the initial orientation of the p^+, p^- pair, at creation.

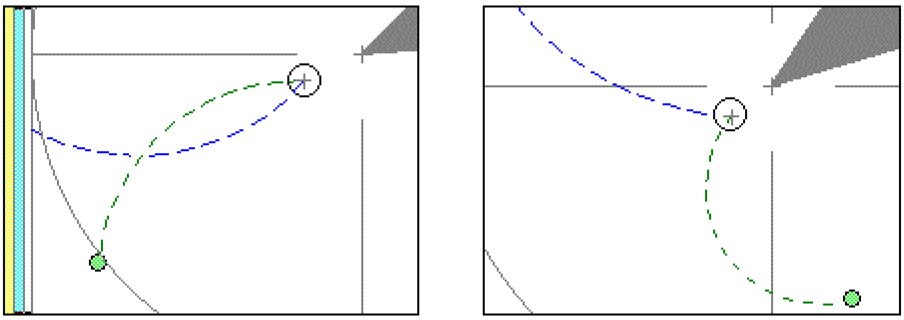


Figure 4.2a 4.2b

4.1 Calculating the K^0 rest mass

To calculate the rest mass of the K^0 (m_K) you first need to determine the momentum of each pion by measuring their radius of curvature. After you have clicked Fire and obtained some pion tracks which you wish to study, you must click the Radius button in order to start Radius measurements. Making these measurements is the same as in the previous exercise - for more information, see section 3.1 '*Making Radius measurements*'. Your points should lie, equally spaced, on a pion track as shown in Figure 4.3. After you have placed your third point the computer will display, in a data box, the radius of curvature (in meters) of a circle which lies through these three points. (Note: the data box displays the radius result of the last three measurements to be taken; upon starting a new measurement the box is cleared ready to display the new result) Take several measurements for each pion track and obtain a mean. As before, incorrect placement of the Radius points will result in *ERROR!* dialogue boxes.

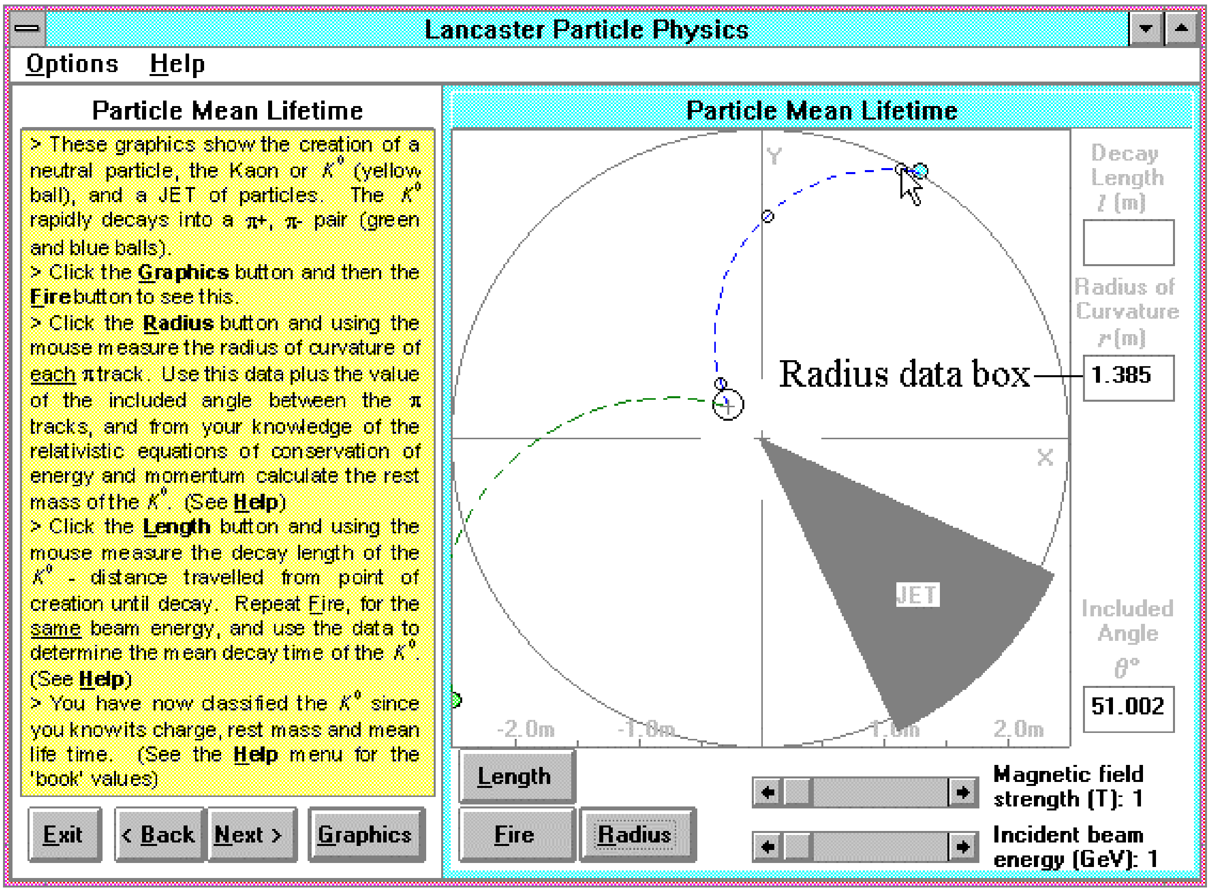


Figure 4.3 'Radius measurements'

Hints: generally, to obtain the best results select: Incident beam energy (E) = 1GeV, Magnetic field strength (B) = 1 T. Fire several times and choose an event where the pions appear to have similar radii; this will ease Radius measurements. Take <u>three</u> or more Radius measurements on <u>each</u> pion track, on the <u>same</u> firing sequence, and obtain a mean radius for each pion.

<u>Note</u>: If the beam energy E is increased this will increase the energy of the K^0, resulting in one or both of the pions having large momentum and producing a large radius of curvature, which will be difficult to measure accurately. Conversely if the magnetic field B is too strong this will result in one or both of the pions having a small radius of curvature which, again, is difficult to measure accurately. There will, of course, be some variation around these optimum settings and it is up to the student to experiment, to find those settings which give the most accurate results.

To calculate the rest mass of the K^0 is not as simple as in the previous exercise. We have to deduce its mass from the measured properties of the pions. Therefore, as expected, equation 4.1 for the K^0 rest mass (m_K) consists of terms relating solely to the pions. (Equation 4.1 is discussed in detail at the end of this section)

$$m_K c^2 = \sqrt{2m_p^2 c^4 - 2 p_1 p_2 c^2 \cos q + 2E_1 E_2} \qquad (4.1)$$

where p_1 and p_2 are the respective pion momenta. They are calculated by taking your radius measurements and substituting them into equation 4.2 (which is the same as equation 3.3 derived in section 3)

$$p_p = 0.3 B r_p \qquad \text{(units: GeV/c)} \qquad (4.2)$$

q is the included angle between the direction of motion taken by each pion, as shown in Figure 4.4. Its value is displayed in a data box after each firing sequence.

E_1 and E_2 are the respective pion energies. They are calculated by using your values for p_1, p_2 and substituting them separately into equation 4.3 which is similar to equation 3.6 (m_π = 0.140 GeV/c^2, as determined in the last exercise).

$$E_p = \sqrt{p_p^2 c^2 + m_p^2 c^4} \qquad (4.3)$$

Substitute your values for p_1, p_2, E_1 and E_2 into equation 4.1. You should yield a result for the neutral kaon rest mass which is close to the 'book' value given below

Neutral kaon rest mass (m_K) = 0.497 GeV/c^2

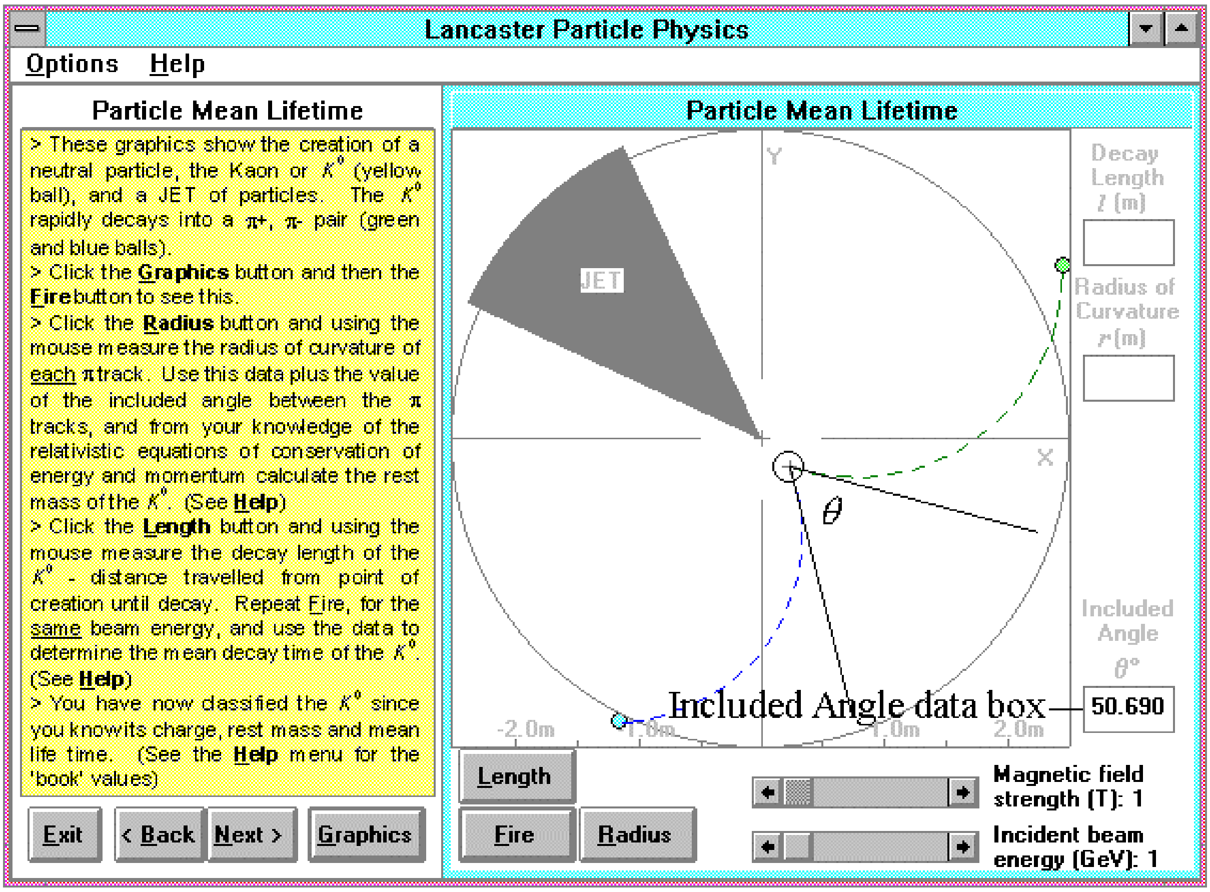


Figure 4.4 showing included angle θ

Lets do an example:

(Remember: $m_\pi = 0.140$ GeV/c^2, so $m_\pi c^2 = 0.140$ GeV)

If your radius measurements give $p_1 = 0.367$ GeV/c, put $p_1 c = 0.367$ GeV

$$\text{then} \quad E_p = E_1 = \sqrt{0.367^2 + 0.140^2}\ \text{GeV}$$
$$E_1 = \underline{0.393\ \text{GeV}}$$

If your radius measurements give $p_2 = 0.594$ GeV/c, put $p_2 c = 0.594$ GeV

$$\text{then} \quad E_p = E_2 = \sqrt{0.594^2 + 0.140^2}\ \text{GeV}$$
$$E_2 = \underline{0.610\ \text{GeV}}$$

For this example θ = 51.653°. We can now calculate m_K

$$m_K c^2 = \sqrt{2 \times 0.140^2 - 2 \times 0.367 \times 0.594 \times \cos(51.653)^\circ + 2 \times 0.393 \times 0.610}\ \text{GeV}$$
$$m_K c^2 = 0.498\ \text{GeV}$$
$$\therefore \quad m_K = \underline{\underline{0.498\ \text{GeV} / \text{c}^2}}$$

This is very close to the 'book' value given above. By repeating the measurement you should obtain a better estimate.

4.1.1 Summary of calculation

Step 1: Select a suitable pair of pion tracks and note down all the data box and scroll bar settings.

Step 2: Take three or more Radius measurements for each pion track and obtain a mean value for both.

Step 3: Substitute your mean radius measurements, separately, into equation 4.2 and determine the momentum for each pion in GeV/c.

Step 4: Substitute your values for the pions momentum, separately, into equation 4.3 and determine their energies in GeV.

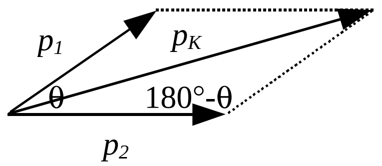
Step 5: Substitute all your values, for the pions momentum and energy, into equation 4.1 and obtain the kaon's rest mass in GeV/c^2.

Aside: The origin of equation 4.1, for the K^0 mass

The mass of the K^0 is given by the squares of its total energy

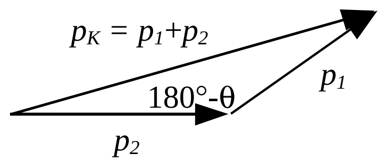
$$E_K^2 = m_K^2 c^4 + p_K^2 c^2$$
$$\text{so}\quad m_K^2 c^4 = E_K^2 - p_K^2$$

We determine E_K and p_K from the energy and momentum of the p^+, p^- pair. Therefore $E_K^2 = (E_1 + E_2)^2$ *i.e.* simple addition of the pion energies. The total momentum squared (p_K^2) can be calculated by applying the cosine rule to the triangle of vectors shown below



parallelogram of vectors

Equivalent to:



triangle of vectors

where p_1 and p_2 are the respective pion momenta.

Hence $$p_K^2 = p_1^2 + p_2^2 - 2p_1 p_2 \cos(180 - q)$$

so that $$m_K^2 c^4 = (E_1 + E_2)^2 - \left(p_1^2 + p_2^2 + 2p_1 p_2 \cos q\right)c^2$$

Expand this equation and rearrange. Remember that $E_1^2 - p_1^2 c^2 = E_2^2 - p_2^2 c^2 = m_p^2 c^4$.

Show that

$$\underline{\underline{m_K^2 c^4 = 2m_p^2 c^4 - 2p_1 p_2 c^2 \cos q + 2E_1 E_2}}$$

4.2 Calculating the K^0 mean Lifetime

The K^0 mean lifetime is the average time it exists from creation, when the incident electron and positron annihilate, until decay when the K^0 degenerates into a p^+, p^- pair. Particle decay is a random process that leads to an exponential distribution of observed decay times. Instead of measuring these decay times, which in reality are exceedingly small, we can measure the decay lengths of the K^0 - which are recorded by the detector. This is the distance travelled by the K^0 from creation, at the origin of the xy-view where the electron-positron annihilation takes place, to the decay point where the pions begin their trajectories - called the 'vertex'. Therefore, we can determine the K^0 mean decay length (L) and then use this to calculate the K^0 mean decay time (τ), for a particular beam energy.

You can estimate a value for L by measuring many decay lengths (l), for the <u>same</u> Incident beam energy, and calulating the mean (see later, section 4.2.1 'Making Length measurements').

Because the K^0 travels at speeds (v) approaching that of light, its properties are affected according to special relativity. Its mean decay time τ, as measured by us in the (stationary) laboratory, is lengthened by the Lorentz factor, $g = 1/\sqrt{1-b^2}$, where $b = v/c$. Hence, the K^0 mean lifetime becomes

$$t_{\text{laboratory}} = g\,t \tag{4.4}$$

The mean distance travelled in the laboratory by the K^0 is equivalent to its mean decay length, L, which can now be expressed as

$$L = v t_{\text{laboratory}}$$

Using equation 4.4, this becomes

$$L = vg\,t$$

$$\text{or} \quad L = bcg\,t \tag{4.5}$$

With, $bg = \sqrt{g^2 - 1}$, we can rewrite equation 4.5 as

$$L = \sqrt{g^2 - 1} \cdot ct \tag{4.6}$$

rearranging equation 4.6 gives

$$t = L/(\sqrt{g^2 - 1}\,.c) \qquad (4.7)$$

You can use equation 4.7 to calculate the mean decay time τ, using your value for *L*. However we also need to determine the Lorentz factor, γ: In the annihilation the K^0 takes half of the available energy; which is equal to the incident beam energy *E*. Therefore, γ for the K^0, can be determined using equations 4.8 and 4.9 for the energy of a particle travelling at relativistic speeds

$$\text{generally,} \qquad E = g\, m_0 c^2 \qquad (4.8)$$

$$\text{or,} \quad g = E/m_K c^2 \qquad (4.9)$$

where m_K is the K^0 rest mass which you determined in the first part of the exercise, *c* is the speed of light and the value of *E*, you chose yourself.

Lets do an example:

Remember $m_K c^2 = 0.497$ GeV. If *E* was set at 10 GeV

$$\text{then} \quad g = \frac{10\,\text{GeV}}{0.497\,\text{GeV}} = 20.1$$

If your length measurements give a mean decay length *L* = 0.933m, then we can now calculate τ

$$t = \frac{0.933\,\text{m}}{\sqrt{20.1^2 - 1} \times 3 \times 10^8\ \text{ms}^{-1}}$$

$$\therefore \quad t = \underline{\underline{1.55 \times 10^{-10}\ \text{s}}}$$

This is close to the 'book' value given below. By taking more Length measurements you may obtain a better estimate.

Neutral kaon mean decay time (τ) = 0.89×10^{-10} s

4.2.1 Making Length measurements

To start Length measurements, click Fire to obtain a K^0 decay then click the **Length** button. (Note: to take measurements you must click the Length button on each new Firing sequence). Measurements are made in a similar way to the Radius measurements: position the mouse pointer on the start, or end points, of the K^0 decay path and click the left-hand mouse button; a circle will be drawn at this point and its centre co-ordinates stored by the computer. Repeat this procedure twice to display two circles (Note: only two points are required for the Length measurement). The computer will then display, in a data box, the distance (in meters) between these two points. If you have chosen your points carefully, as shown in Figure 4.5, this distance should correspond to the kaon decay length.

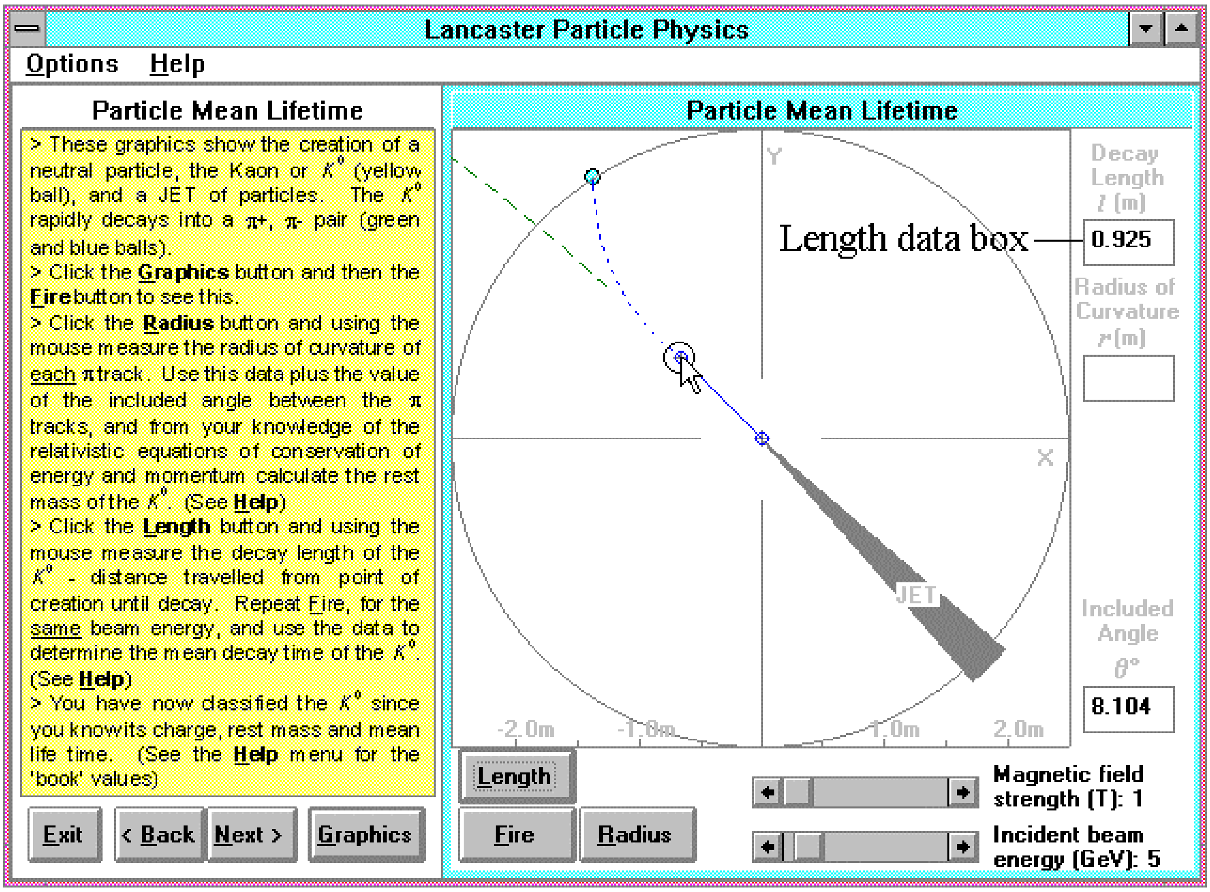


Figure 4.5 'Length measurement'

It is up to you to position the Length measurement circles correctly, poor positioning will result in an inaccurate value for *l* producing an inaccurate value for the kaon's mean decay length. As a guide to correct point placement the 'ghost' K^0 (white ball) has cross-hairs positioned at its centre; by placing one of your points on these cross-hairs and your other point at the origin of the xy axis, you should always achieve an accurate measurement (see Figure 4.5). However, if your points are either too far apart or too close together an *ERROR!* dialogue box appears telling you to try again, as shown in Figure 4.6. Click the OK button and this time take care to place your points on the start and end points of the K^0 decay path.

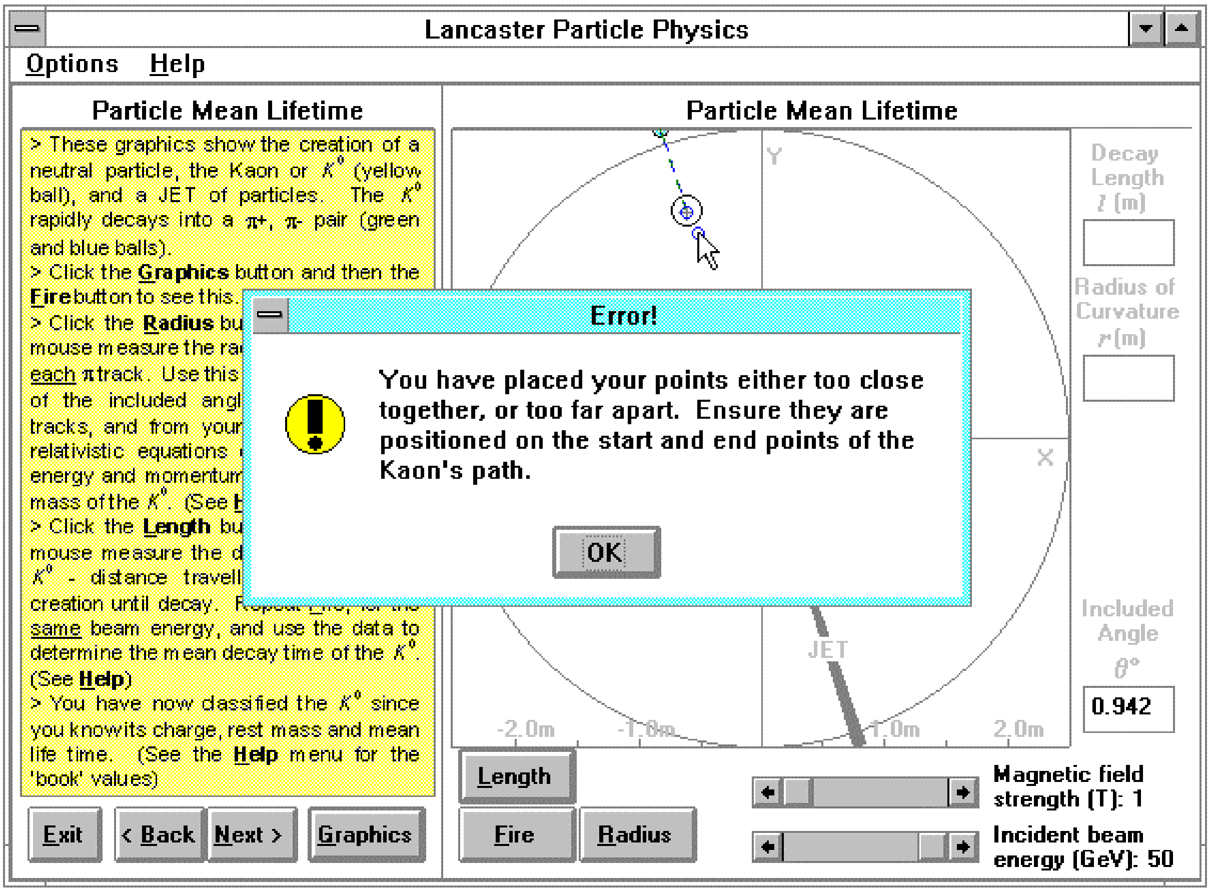


Figure 4.6 'Error box'

Hint: before starting your Length measurements it is advised to select a large Incident beam energy (E), which you should note and keep the same throughout all your measurements. You will find, in accordance with equation 4.5, that larger beam energies result in larger decay lengths, which can be measured with greater accuracy.

Note: in order to obtain a resonable estimate for L, it will be necessary for you to take a minimum of 10 Length measurements for the same beam energy. Because of the relative ease in obtaining accurate measurements and the large number required, you don't need to repeat measurements, unless you cause an *ERROR!* box to appear.

4.2.2 Summary of calculation

Step 1: Select a large E, note its value and keep the same throughout all your measurements.

Step 2: Click Fire and take Length measurements on each firing sequence. You will need a minimum of 10 measurements. Calculate a mean decay length, L.

Step 3: Calculate γ using equation 4.9, with your choice of E. Substitute γ and L into equation 4.7 to determine the K^0 mean decay time, τ.

(Note: with 10 measurements the accuracy is limited)

Additional Exercise

Here is an alternative method for calculating L which, while taking more time, should yield a better estimate for τ. It is expected that the teacher will guide the students through the more complicated parts of this procedure.

1) Take a minimum of 30 Length measurements for the same beam energy. Plot a histogram of the distribution of K^0 decay lengths, $N(l)$, which should follow the exponential relation below

$$N(l) = N(0)\exp(-l / L) \qquad (5.0)$$

2) Transform the histogram into a linear graph by taking natural logs of both sides of equation 5.0. The slope of this straight line graph gives $1/L$. You can now use this value of L to determine τ, as before. Compare your value for τ with your earlier estimate, you should find that it is closer to the ‘book’ value. Why is this?

The reason for this is due to the simulation and not you. The software imposes an upper and lower ‘cut-off’ on the K^0 decay lengths, so that they always occur within the graphics viewport, and not at distances too small to measure. This produces a biased value for L if you calculate a mean, because we have missed out the large and small values of l.