3 Charged Particle Motion in a Magnetic Field

When you have completed the ‘Particle Annihilation’ section and read all the text (especially section 2.2), click the Next button in the Particle Annihilation text window to move onto the next section. Or select ‘Motion in a magnetic field’ from the Options menu. Either way the text window for ‘Motion in a magnetic field’ will appear.

Aim: to identify different charged particle-antiparticle pairs, by studying their trajectories in a magnetic field.

Click the Graphics button in the text window to display the graphics window as shown in Figure 3.1. As explained in the previous section we are now looking at a circular cross-sectional view of the point where the collisions take place, at the LEP accelerator at CERN, which we have labelled as the *xy*-plane. The incident electron and positron beams travel along the axis of the accelerator, *i.e.* into and out-of the screen; they meet and annihilate in the centre of this view, at the origin of the *x*, *y* axes. In the real accelerator this view is several meters across; see scale at the bottom of the graphics viewport.

Click the Fire button in the graphics window. This causes an annihilation as indicated by the *POW!* and two charged particles are created. Their trajectories (dashed lines) are curved by the uniform magnetic field applied along the *z* axis, *i.e.* coming out-of the screen towards you! (see Figure 3.1 and Figure 2.7).

Scroll bars, positioned at the bottom of the graphics window, allow the Incident beam energy (E) and the Magnetic field strength (B) to be changed and set, prior to firing. The allowed ranges are E = 1 to 50 GeV; B = 1 to 5 Tesla. The computer selects, at random, from four charged particle pairs, shown below with their rest masses

1) muon+, muon– (m^+, m^-)	= 0.106 GeV/c^2
2) pion+, pion– (p^+, p^-)	= 0.140 GeV/c^2
3) kaon+, kaon– (K^+, K^-)	= 0.494 GeV/c^2
4) proton-antiproton (p, $\bar{p}$)	= 0.938 GeV/c^2

Hence, each firing sequence can result in a different charged particle pair being formed. The particles are colour coded; it is your task to identify them by determining their rest masses and comparing your answer with the 'book' values, given above. To see how this is done we must first discuss which units to use and study the relativistic equation connecting total energy and momentum.

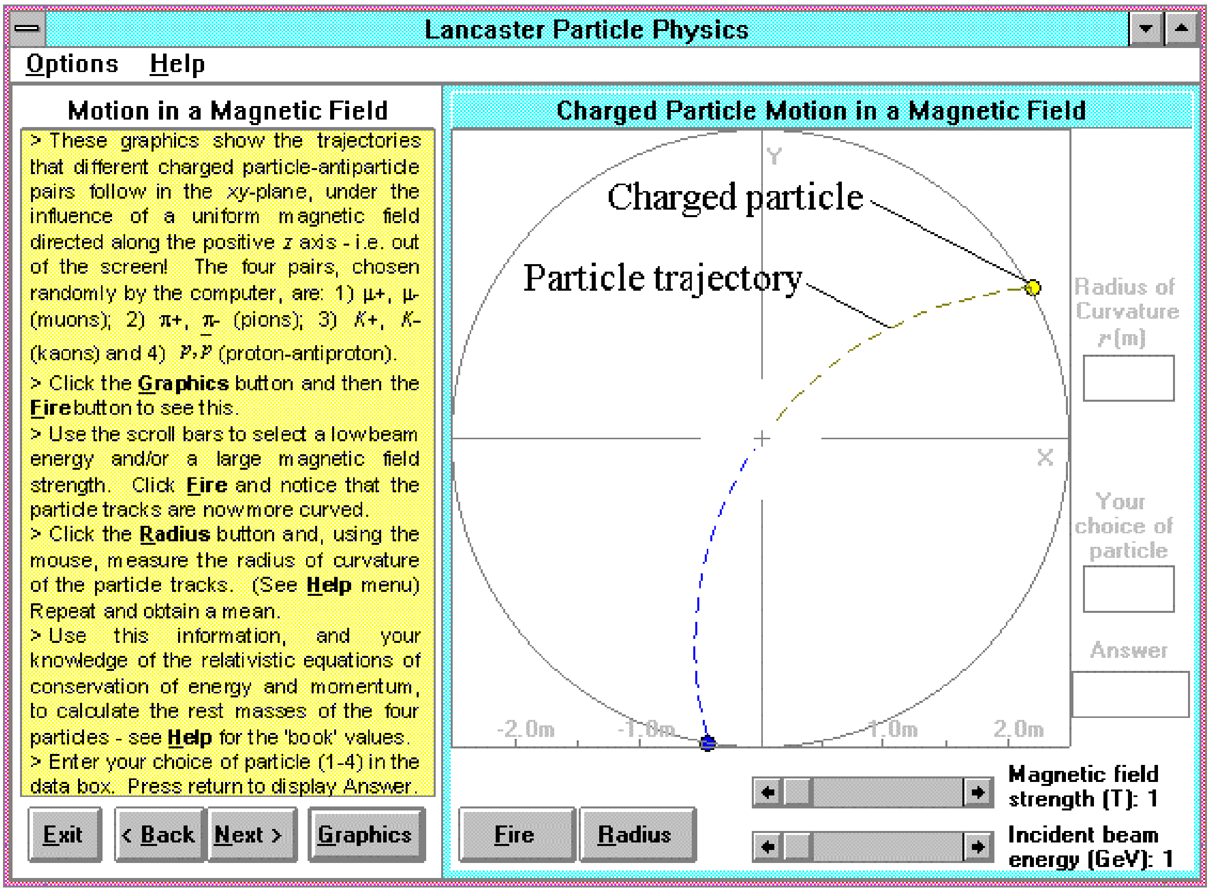

Figure 3.1 'A typical firing sequence'

The charged particles experience a force given by equation 3.1

$$F = BQv \tag{3.1}$$

because B and v lie in perpendicular directions this force will cause the particles to travel in circular trajectories. Hence, for circular motion, equation 3.1 gives

$$\text{mass} \times \text{centripetal acceleration} = \text{centripetal force}$$

$$m\frac{v^2}{r} = BQv$$

$$\therefore \quad mv = BQr$$

$$\text{or} \quad p = BQr \qquad \text{(units: Kg. m. s}^{-1}\text{)} \quad (3.2)$$

where p is the particle momentum and r is the radius of curvature of its trajectory. Therefore if we know B and Q we need only determine r in order to calculate the particle's momentum. However, in order to get a useful answer we need to convert the momentum, at present in m.k.s units to GeV/c units.

The unit of Q is Coulombs (C), B is Tesla (T) and r is meters (m). If we multiply both sides of equation 3.2 by the speed of light, $c = 3\times10^8$ ms^{-1}, then the units are now in Joules because: [Momentum × Speed] = [Energy]

$$pc = BQrc \qquad \text{(units: Joules (J))}$$

One electronvolt, 1 eV = 1.6×10^{-19} J

or, expressed another way, 1 J = $(1/1.6\times10^{-19})$eV.

Therefore the units of the equation, above, can be converted to eV as follows

$$pc = \frac{BQrc}{1.6\times10^{-19}} \qquad \text{(units: Electronvolts (eV))}$$

but Q is equal to the charge on the particle moving in the magnetic field. For this exercise Q is equal to the charge on one electron or proton = $1.6\text{x}10^{-19}$ C. Therefore the equation above reduces to

$$pc = Brc \qquad \text{(units: eV)}$$

by substituting in the value for c, on the right hand side, we get

$$pc = Br\,3\times10^{8} \qquad \text{(units: eV)}$$

or, because 1GeV = 1×10^9 eV

$$pc = 0.3\,Br \qquad \text{(units: GeV)}$$

Finally, by expressing the units in terms of c we obtain:

$$p = 0.3\,Br \qquad \text{(units: GeV/c)} \quad (3.3)$$

(Note: Even though we have derived this formula using classical mechanics it is also valid for the HEP case, *provided we use the relativistic momentum,* $p = g\, m_0 n$)

Hence the units of momentum are GeV/c, *B* still has units of Tesla, *r* of meters. As a summary:

1) Energy has units **GeV**

2) Momentum **GeV/c**

3) Mass **GeV/c²**

(These are very convenient units in which to measure energy, mass and momentum when working with particles of very light mass accelerated to very high energies. For example, it is much easier to remember that the rest mass of the proton is roughly 1 GeV/c², than it is to remember that its value is 1.67×10^{-27} Kg !)

You should use equation 3.3 to determine the momentum of your particle by making measurements of its radius of curvature and by using the value of *B* set by you, prior to firing, with the scroll bar in the graphics window (see Figure 3.1). Making radius measurements is covered in section 3.1, below; for the present we will continue with the calculation to determine the particle rest mass.

When you have calculated your particle's momentum you can use the important relativistic relation connecting total energy and momentum, to calculate its rest mass

$$E^2 = p^2 c^2 + m_0^2 c^4 \qquad (3.4)$$

where E is the total energy of the particle (kinetic plus rest energy) and m_0 is the rest mass. We can see that for a particle at rest (p = 0) this equation reduces to the familiar, $E = m_0 c^2$.

Because energy is conserved, the total energy <u>before</u> annihilation equals the total energy <u>after</u> annihilation. In our case, before annihilation, the electron and positron each have an energy equal to the beam energy, therefore the total energy of the system is twice this amount. After annihilation, when a particle-antiparticle pair are formed, each particle takes half of the available energy: equal to the original beam energy. Therefore in equation 3.4, E = Incident beam energy. This can be set by you, prior to firing, using the scroll bar in the graphics window (see Figure 3.1).

We now know both the energy and momentum of each particle in the charged particle pair. Equation 3.4 can be rearranged to determine their rest masses

$$m_0 = \frac{1}{c^2}\sqrt{E^2 - p^2c^2} \qquad (3.5)$$

Lets do an example:

If the Incident beam energy was set at 1 GeV, put E = 1 GeV

If your radius measurements give p = 0.873 GeV/c, put pc = 0.873 GeV

then $m_0 = \sqrt{1^2 - 0.873^2}\ \mathrm{GeV/c^2}$

$m_0 = \underline{\underline{0.488\ \mathrm{GeV/c^2}}}$

This is very close to the mass of a kaon (K^+, K^-), see the table on page 16. By repeating the measurement you could confirm this - see section 3.1 *'Making Radius measurements'*.

3.1 Making Radius measurements

After you have clicked Fire and obtained some particle tracks that you wish to study, you must click the 

button in order to start Radius measurements.

Measurements are made by positioning the mouse pointer on a particle track and clicking the left-hand mouse button; a circle will be drawn at this point and its centre co-ordinates stored by the computer. Repeat this procedure three times to display three circles. The computer will then display, in a data box, the radius of curvature (in meters) of a circle which lies through these three points (remember three points uniquely define a circle). If you have chosen your points carefully they should lie, equally spaced, on the particle track and the displayed radius value should correspond to the radius of the particle track (see Figure 3.2b).

It is up to you to position the Radius circles correctly, poor positioning will result in an inaccurate value for *r* making it difficult, if not impossible, to determine the particle's rest mass. Figure 3.2a shows poor circle placement and Figure 3.2b shows the correct placement.

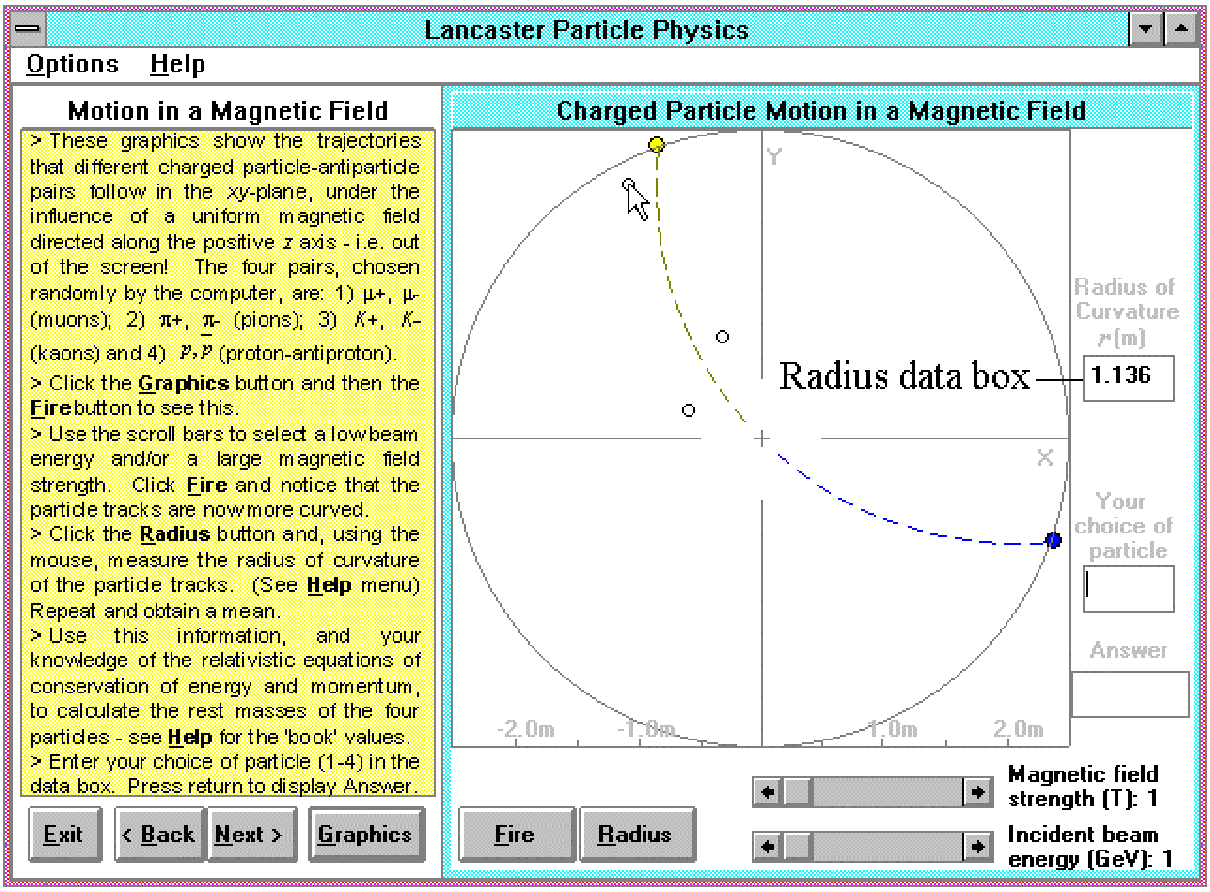

Figure 3.2a Poor circle placement

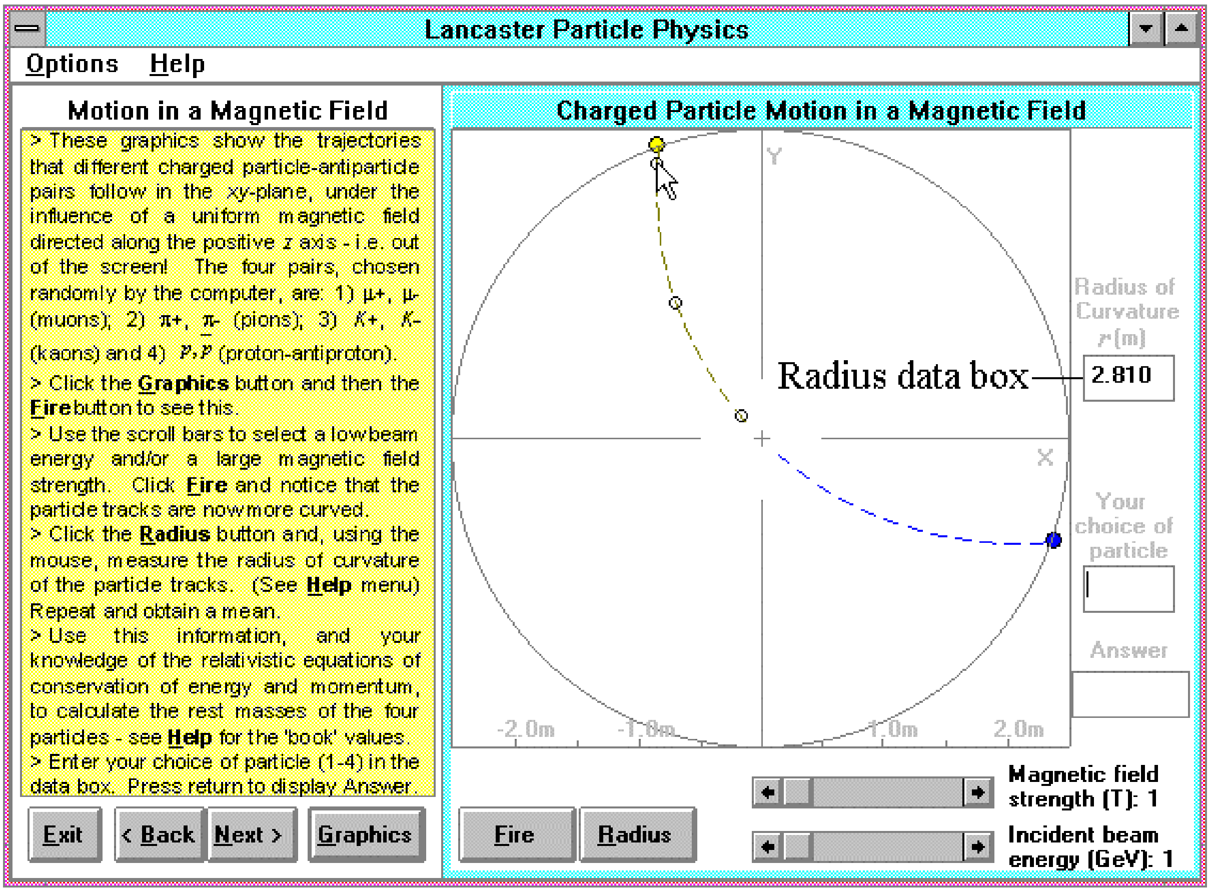

Figure 3.2b Correct circle placement.

There are two *ERROR!* dialogue boxes that can appear when you are placing your Radius points. If your points are too <u>close</u> together then an error box appears, as shown in Figure 3.3a, telling you to try again, this time spacing your points evenly along the track to obtain a more accurate value for your radius. If you have chosen a large incident beam energy then the particle tracks can often appear as straight lines and your points may lie in a line, giving an <u>infinite</u> radius of curvature! If this happens an error box will appear, as shown in Figure 3.3b, advising you to try again on a new particle track, this time with a lower beam energy and/or a larger magnetic field strength. Either way this should produce a track that is more curved - can you see why? Increasing the magnetic field increases the force which pushes the charged particles into a circular path. Reducing the beam energy reduces the new particles linear momentum and so, according to equation 3.3, the radius of curvature is reduced.

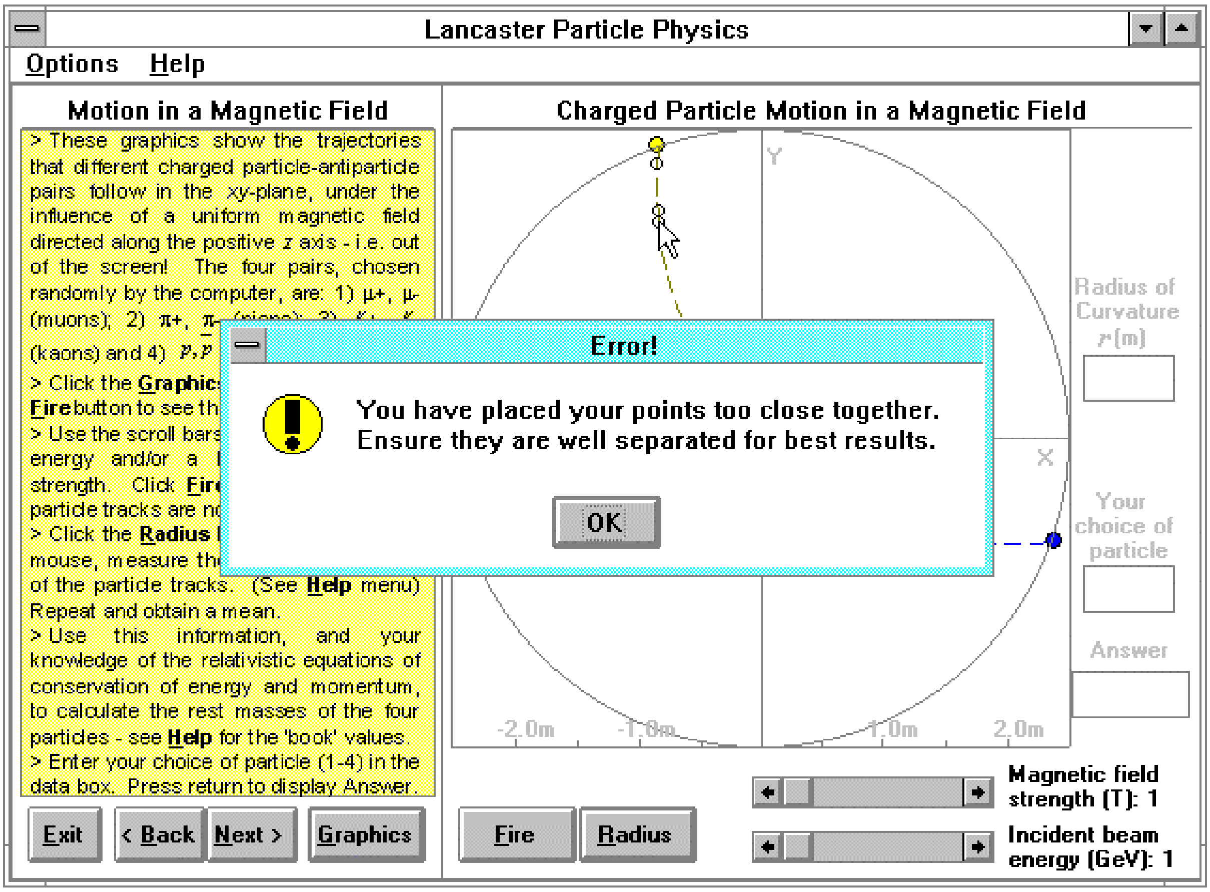

Figure 3.3a Error boxes

Even when you have placed your points correctly, it is still advisable to repeat the measurements on the same particle track, in order to obtain a mean radius for the same incident beam energy and magnetic field settings. To start a new measurement, position and click the mouse a fourth time, this will then become the first point of a new Radius measurement and the previous points will be erased automatically. Select a further two points, so that you again have three circles, and the new radius value will be displayed in the data box. Make a note of your *r* values each time and calculate a mean.

Hints: generally, to obtain the best results select: Incident beam energy (*E*) = 1GeV, Magnetic field strength (*B*) = (1 or 2) T and take three or more Radius measurements on the same particle track. As discussed above if *E* is too large the radius is too large, while if *B* is too large the radius will be too small making radius measurements, in both instances, inaccurate. With practice you will find suitable settings which result in reliable particle identification from their rest masses.

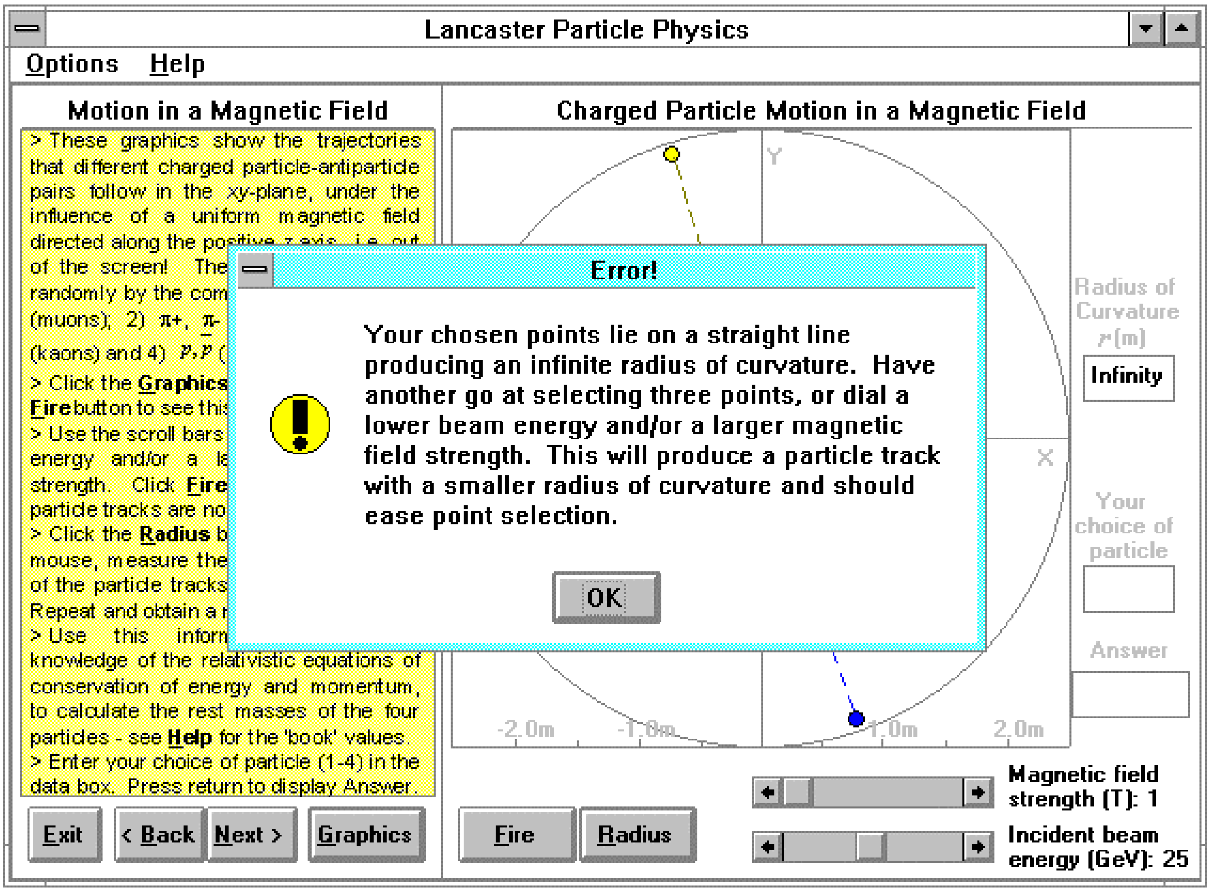

Figure 3.3b Error boxes

Warning: Those particles with smaller rest masses will be difficult to determine, because their rest mass energy ($m_0 c^2$) is much less than their kinetic energy even at the lowest beam energy of 1 GeV. In terms of equation 3.5 this means there is very little difference between the two terms in the square root, and the momentum must be determined very accurately in order to obtain a meaningful answer: *i.e.* if m_0^2 comes out negative you are not positioning your cursor accurately enough.

To see if you have correctly identified the particle, from your rest mass calculations, enter the corresponding number (1 to 4 from the list on page 16) of your choice of particle into the data box containing the flashing cursor and press return. This will cause the name of the correct particle to be displayed (in red) in the Answer box. For comparison, the program will also convert your particle number into the corresponding particle name, as shown in Figure 3.4. Note: if you enter an invalid entry such as a character, instead of a number, then an *ERROR!* dialogue box will appear. Alternatively, if you enter a number outside the 1 to 4 range or one that is not an integer then two separate *TYPING ERROR!* dialogue boxes will appear. Click the OK button to clear the dialogue boxes. This will also cause the data box to clear, after which you can enter your choice of particle.

(Note: the data entry box, for your choice of particle, is only enabled after you have taken your three Radius measurements. The Answer will only be displayed after you have determined a type of particle and entered, correctly, its corresponding integer number into the data box and pressed return.)

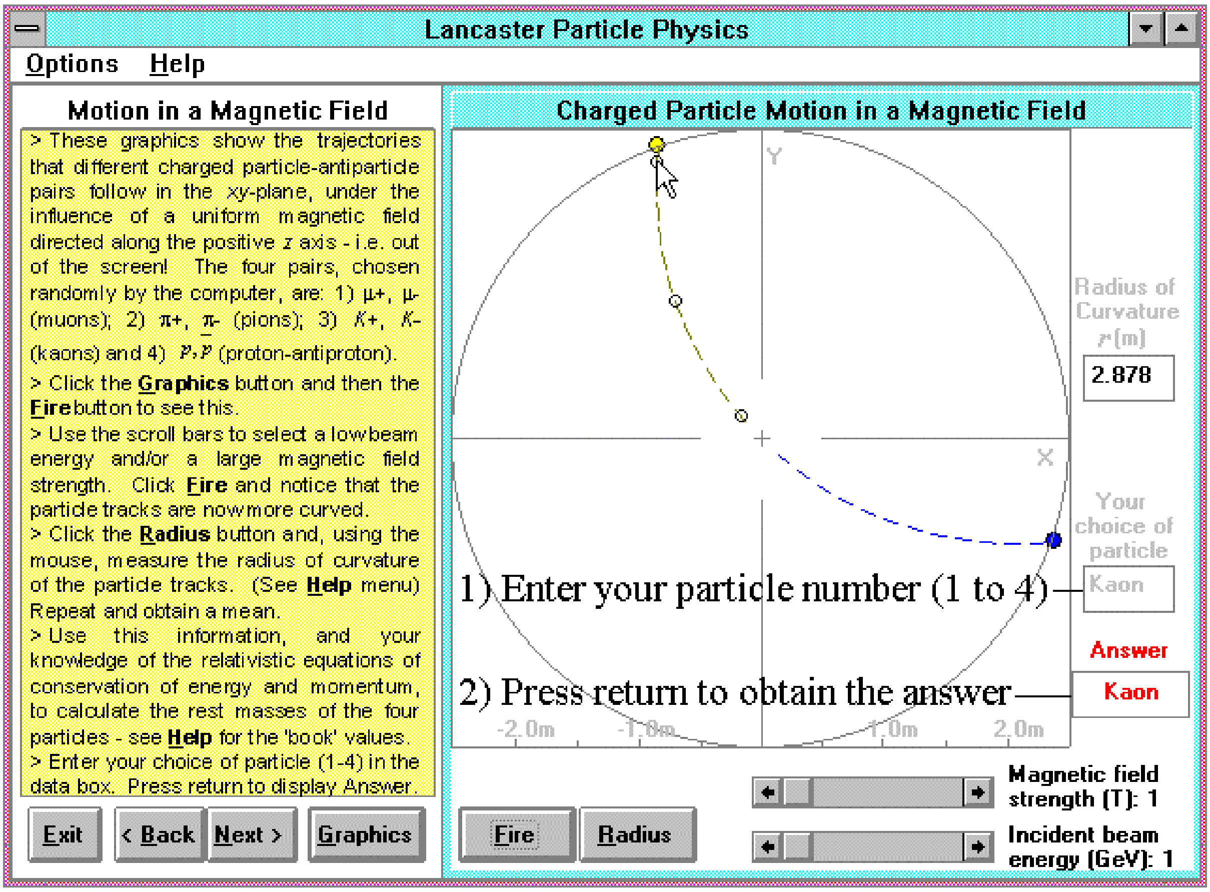

Figure 3.4

3.2 Summary of calculation

Step 1: Select your *E* and *B* settings to produce a track which is easy to measure.

Step 2: Take three or more Radius measurements on the same particle track and obtain a mean value.

Step 3: Substitute your mean radius measurement into equation 3.3 and determine the momentum of your particle.

Step 4: Substitute your momentum value into equation 3.5 and obtain the particle rest mass.