2 Particle Annihilation and Creation

When you have completed the 'Billiards' exercise click the Next button in the Billiards text window to move onto the next section. Or select 'Particle Annihilation' from the Options menu. Either way the text window for 'Particle Annihilation' will appear.

Aims:

1) To compare two methods of particle creation: Stationary target versus Colliding beams. In a collision between a moving particle and one that is stationary, most of the energy is 'wasted' in conserving momentum and cannot go into creating mass. However, if the particles collide head-on all the energy can be used to create new particles. Hence colliders use energy much more efficiently. This is dramatically illustrated in this exercise.

2) Introduction to the following options and exercises, which are all concerned with electron-positron annihilations at the CERN accelerator.

Physicists accelerate particles up to high energies and collide them together in order to study their internal structure. The higher the energy the smaller the distance scale that can be resolved. To understand this consider the following: electrons in atoms require a few electronvolts (eV) to be excited to a higher energy state, before they return to their previous state and radiate their excess energy as photons in the visible spectra. To ionise an atom, *i.e.* to remove an electron from an atom requires a similar amount of energy. To excite protons and neutrons in a nucleus (smaller scale) requires a few MeV since the nucleus is a more strongly bound structure. It turns out that protons and neutrons are made up of quarks which are more tightly bound still. Hence energies in the GeV range (10^9 eV) are required to investigate such fine detail and we are now in the realm of High Energy Physics - HEP.

Because the particle energies are so large in HEP, we have to use the correct relativistic equations to describe the collisions. If particle collisions are sufficiently energetic, new particles can be created. Remember there is an equivalence between mass and energy given by Einstein's relation $E = m_0 c^2$. We can use the laws of conservation of momentum and conservation of total energy to analyse HEP particle collisions. The expression for the total energy of a particle with rest mass m_0 and momentum p is

$$E^2 = p^2 c^2 + m_0^2 c^4 \qquad (2.1)$$

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Physicists also collide particles and their antiparticles. This is the case at the Large Electron Positron machine (LEP) at CERN; here electrons and their antiparticles, positrons, collide together at high energies. A positron has the same mass as the electron but with a positive charge as opposed to the electron's negative charge. When an electron and positron collide they annihilate each other leaving a *'blob'* of energy in the form of a virtual photon. The energy of the photon is equal to the sum of the total energies of the electron and positron. After a very short interval of time the virtual photon materialises into a new particle antiparticle pair. The type of particle that can be produced depends on the energy of the initial collision.

The role of large accelerators, such as LEP, can now be recognised: in order to study the fine structure of matter, which is revealed in the production of new heavy particles, incident beams of electrons and positrons need to be accelerated up to enormous energies such that, at annihilation, sufficient energy is available for new particle creation.

2.1 Stationary Target and Colliding Beams

There are two methods for annihilating and creating particles in particle accelerators, we shall compare both in the example below.

We illustrate this by studying the following process:

$$e^+ + e^- \to m^+ + m^-$$

The (muon) m particles are each about 200 times the mass of an electron, so most of the kinetic energy of the electron and positron must be used to create them. What is the most efficient way of doing this? We take two cases:

1) Stationary target

2) Colliding beams

In case (1) the positron e^+ is fired at a stationary electron (in practice the electron would be bound in an atom with kinetic energy of a few eV, which is negligible compared with the high energies involved in particle physics) whilst in case (2) the e^+ and e^- are fired towards each other with equal and opposite momenta.

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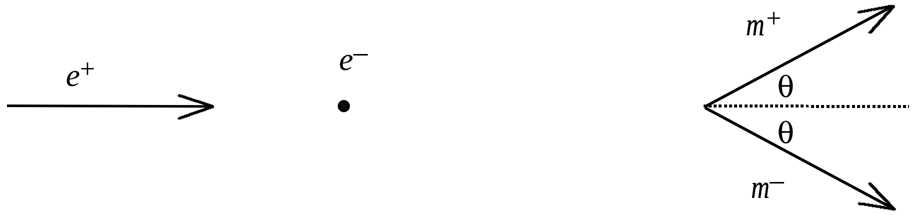
1) Stationary target



Before ***After***

Figure 2.1

The incident positron (e^+) has momentum p_e and total energy E_e and collides with a stationary electron (e^-) thereby annihilating to produce a 'virtual photon'. The virtual photon materialises into a new e^+ and e^- pair or any other heavier particle antiparticle pair, such as m^+ and m^-, provided sufficient energy is available. The m^+ and m^- pair must have total momentum equal to p_e in order to conserve momentum in the collision. We make a simplification and assume the pair are created with equal momenta and at the same angle to the positron direction (see Figure 2.1). (This simplification does not affect our final conclusions).

Conservation of momentum in the positron direction gives:

$$p_e = 2 p_m \cos q$$

$$\text{so that} \quad p_m = p_e \; / \; 2 \cos q$$

At the positron beam energy which is just sufficient to create a m^+, m^- pair (threshold energy), $q = 0^\circ$. The m particles then emerge moving in the same direction as the positron beam with momentum

$$p_m = p_e \; / \; 2$$

Now let's see what conservation of total energy gives. The total energy before a collision is $E_e + m_e c^2$ because the target is stationary and possesses only its mass energy. The total energy of each m^+ and m^- is the same because they have equal momentum. We then have

$$E_e + m_e c^2 = 2 E_m$$

$$\text{so that} \quad E_m = \left(E_e + m_e c^2 \right) / \; 2$$

But we must satisfy the relativistic relation between total energy and momentum for each m particle:

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$E_m^2 = p_m^2 c^2 + m_m^2 c^4$

substituting $E_m = \left(E_e + m_e c^2\right) / 2$ and $p_m = p_e\ / 2$ we get after some rearragement:

$E_e^2 + 2\,E_e\,m_e\,c^2 + m_e^2\,c^4 = p_e^2\,c^2 + 4\,m_m^2\,c^4$

but $\quad E_e^2 - p_e^2\,c^4 = m_e^2\,c^4$

so that $\quad 2E_e\,m_e\,c^2 = 4m_m^2\,c^4 - 2m_e^2\,c^4$

and
$$E_e = 2\frac{\left(m_m c^2\right)^2}{m_e c^2} - m_e c^2 \qquad (2.2)$$

This is the positron total energy at which *m* particles are produced with the least possible momentum, it is therefore the threshold energy.

Put the masses into this equation and calculate the threshold total energy for *m* pair production by positrons hitting a stationary target:

$$m_m c^2 = 105 \text{ MeV}$$
$$m_e c^2 = 0.51 \text{ MeV}$$

2) Colliding beams

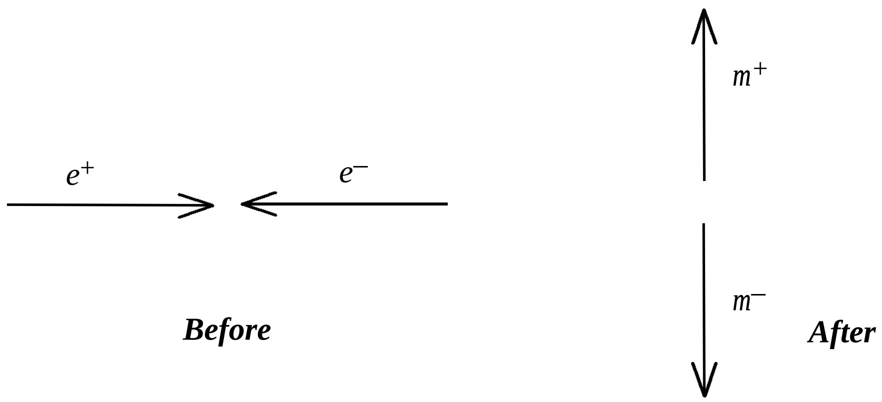

Figure 2.2

The particle antiparticle pair (e^+ and e^-) collide and annihilate to produce a virtual photon with zero momentum and total energy equal to the sum of the e^+ and e^- total energies. The ‘virtual photon’ then materialises into a new e^+ and e^- pair or any other heavier particle antiparticle pair, such as m^+ and m^-, provided sufficient energy is available. If a m^+ and m^- pair is created they will move off with equal and opposite momenta (remember that momentum is conserved and the total momentum is zero). We assume that the pair move at 90° to the e^+ and e^- directions (this simplification does not affect our final conclusions) see Figure 2.2.

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Let the total energy of each positron and electron be E_e, then the total energy before the collision is $2E_e$. After the collision the total energies of each *m* particle are equal since they have equal and opposite momenta. Conservation of total energy in the process then requires:

$$2E_e = 2E_m$$

$$\text{and} \quad E_e = E_m$$

But we have the relativistic relation connecting energy and momentum for each *m* particle:

$$E_m^2 = p_m^2 c^2 + m_m^2 c^4$$

substituting $E_m = E_e$ and rearranging we get:

$$p_m c = \sqrt{E_e^2 - m_m^2 c^4}$$

The expression inside the square root must be positive to give a physically possible momentum. Hence:

$$\boxed{E_e \geq m_m c^2 \qquad (2.3)}$$

The total energy of each electron and positron in the colliding beams must be greater than the mass energy of a *m* particle. The mass of a *m* particle is 105 MeV/c^2 and it follows that:

$$E_e \geq 105 \text{ MeV}$$

Therefore, the "threshold energy" for m^+ and m^- production by annihilation of an electron positron pair is 105 MeV. It is clear that, in this case, <u>all</u> the energy of the colliding particles can be used to create the new particle pair. This gives a threshold energy that is **400** times **less** than in the stationary target case (105 MeV compared with 43 GeV !! In the stationary target case most of the beam energy goes into making the forward momentum of the *m* pair and very little is left over to create the pair). This makes the colliding beams method the natural choice for particle accelerators, as is the case for LEP.

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Muon pair creation, by both the stationary target and colliding beams methods, is simulated in the software. Click on the Graphics button in the text window to display the 'Particle Annihilation' graphics window. On opening the graphics window the default setting of the **Option** buttons is **'Stationary Target'**. Click the Fire button to start the positron on a collision course towards the stationary electron (green and red balls) positioned in the centre of the graphics viewport. They meet in the centre and annihilate, represented by the *POW!*, and form a new e^+ and e^- pair or a m^+ and m^- pair, provided sufficient energy is available. (see Figure 2.3)

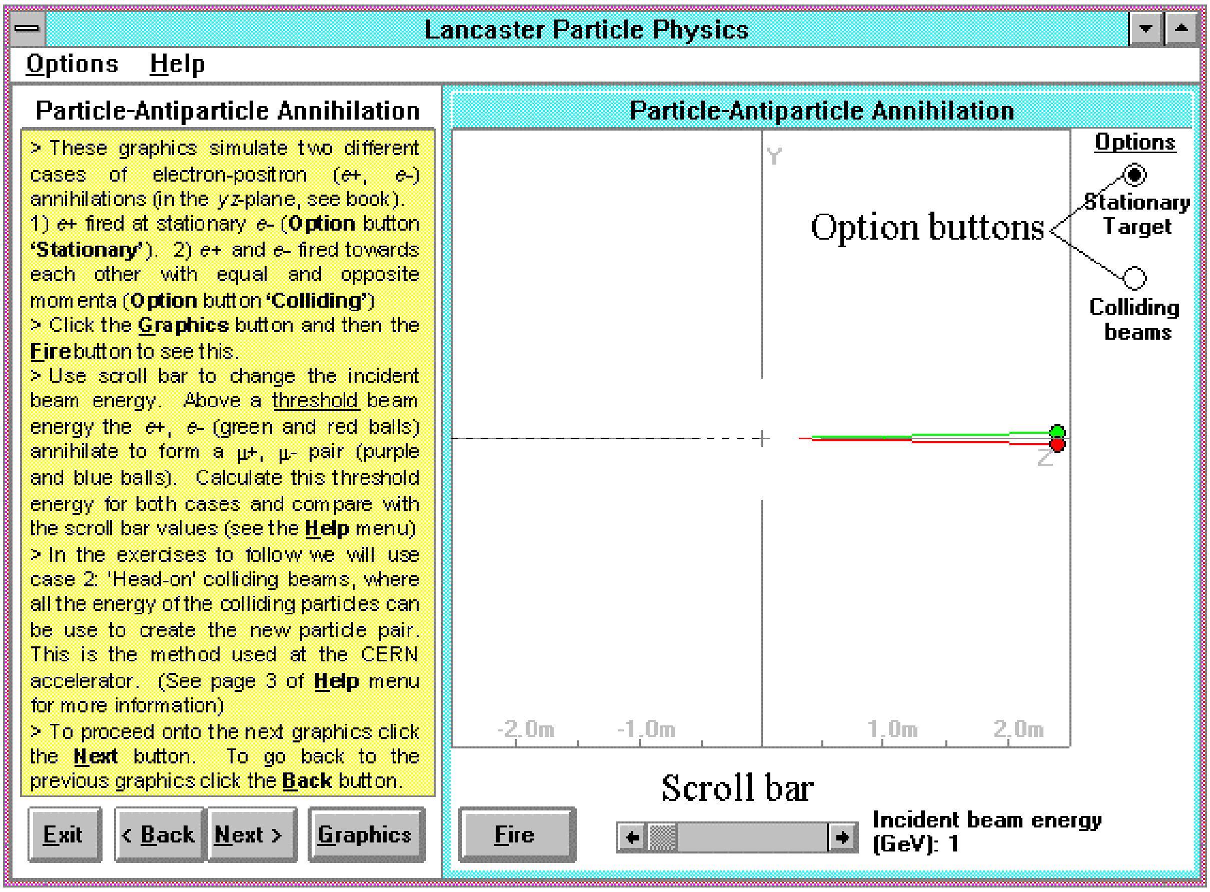

Figure 2.3 'Stationary target'

The positron Incident beam energy can be altered using the scroll bar at the bottom of the graphics viewport (see Figure 2.3). Calculate the threshold energy using equation 2.2 and compare with the scroll bar settings by gradually increasing the beam energy, clicking Fire each time. Above the threshold energy you should notice that the particles change colour, indicating m pair production.

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Select **'Colliding beams'** by clicking on the round **Option** button. Click the <u>F</u>ire button to start <u>both</u> the electron and positron moving on a collision course. Again, they meet in the centre and annihilate, represented by the *POW!*, and form a new e^+ and e^- pair or a m^+ and m^- pair, provided sufficient energy is available. (see Figure 2.4)

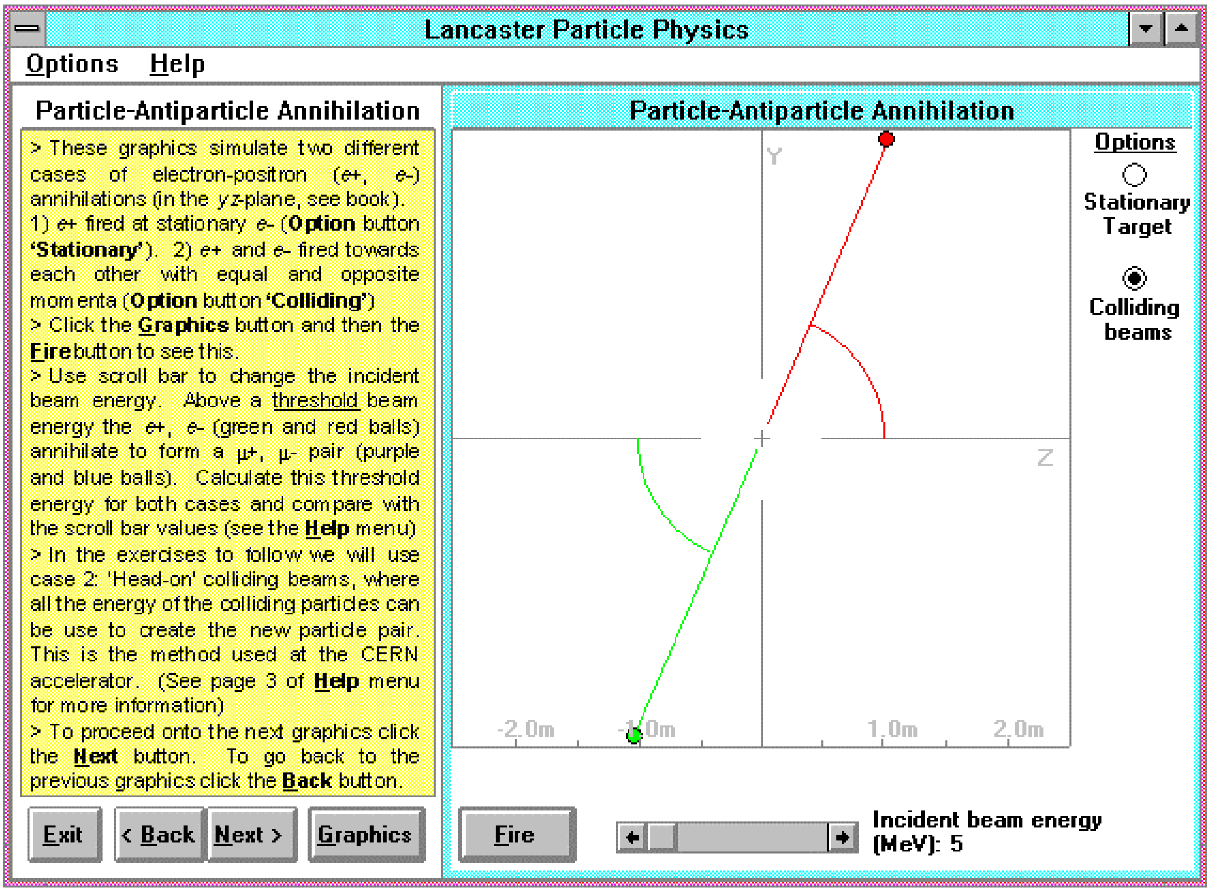

Figure 2.4 'Colliding beams'

Gradually increase the scroll bar settings, clicking <u>F</u>ire each time. This is now the Incident beam energy of <u>each</u> electron and positron. Note the threshold energy, above which *m* pairs are produced and compare with equation 2.3. As expected the threshold energy is drastically reduced compared to that required for the stationary target case.

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2.2 Particle Annihilations at CERN

Particle Annihilations at CERN use the **colliding beams** method. In the simulation for the colliding beams option the graphics represent a 'side' view of the LEP accelerator. Incident beams of electrons and positrons move, in opposite directions, along the axis of the accelerator; defined as along the positive and negative z direction. This can be seen by clicking on the Help page for this option, which is reproduced below in Figure 2.5.

(**Note:** this is a schematic picture. In reality the accelerator has a diameter of 27 Km and a beam pipe diameter of only 10 cm !)

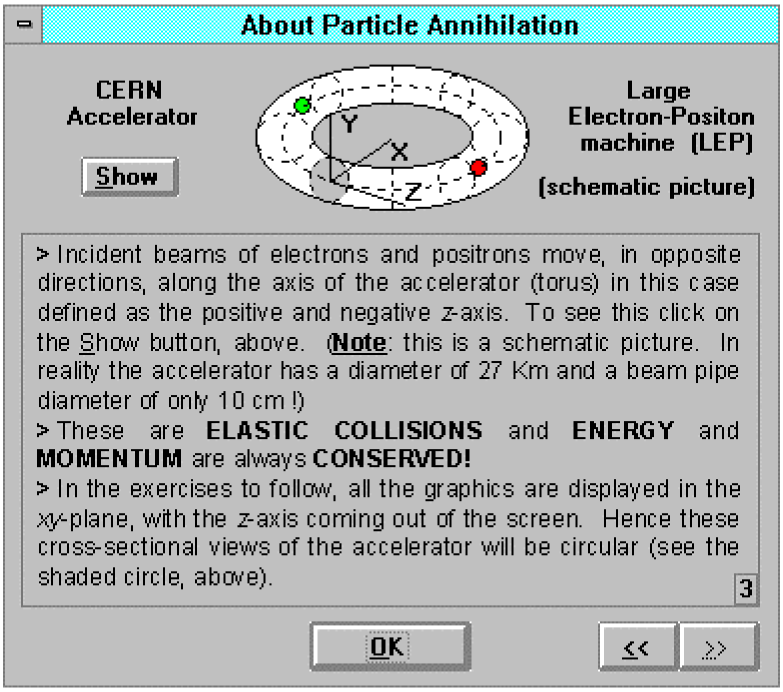

Figure 2.5 'Particle Annihilation' help page

As you can see from Figure 2.5 LEP is rather like a large 'doughnut' (torus) and particle interactions, after annihilation, will inevitably occur in three dimensions. We chart the position of the particles by noting their co-ordinates on three perpendicular axes: X, Y and Z. In all the options/exercises to follow, the graphics are simplified by displaying them in two dimensions. Hence, in the present graphics the new particles appear to be confined to the *yz*-plane. In reality they could also move in the direction of the $\pm x$ axis, which for this orientation is into and out-of the screen.

Notice that the new particles are created with equal, but arbitrary, angles compared to the incident particle direction, before annihilation (see Figure 2.4). This is a consequence of the

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conservation of momentum. Before annihilation the electron and positron have equal but opposite momentum; therefore the total momentum of the system is zero. This must also be true for the total momentum of the system after annihilation, which requires that the new particles also have equal and opposite momentum, causing them to travel in opposite directions.

In the following exercises we will only consider the motion of the new particles when confined in the *xy*-plane, again as a simplification. For the present *yz* view, this will make the new particles appear as if they are travelling along the $\pm y$ axis. Looking into the *xy*-plane will give a circular cross-sectional view of the LEP accelerator, with the *z* axis coming out of the screen (see Figure 2.6).

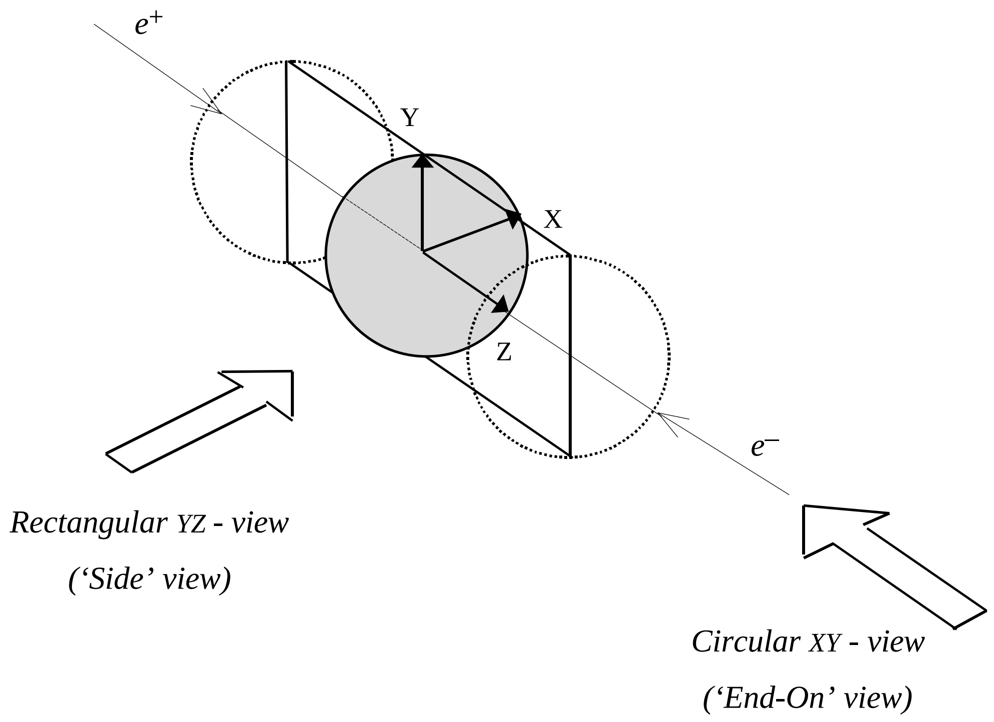

Figure 2.6

As in the real accelerator uniform magnetic fields will be applied along the positive *z* direction, at the point of annihilation *i.e.* at the centre of the detector. This will cause charged particles moving in the *xy*-plane to experience a force, *F*, given by equation 2.4

$$F = B\,Q\,v \tag{2.4}$$

where *B* is the magnetic field strength, *Q* the particle charge and *v* the particle velocity, which is directed within the *xy*-plane and in a direction following Fleming's rule (see Figure 2.7).

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In this case B and v are perpendicular and the force F is always perpendicular to both B and v. F will cause the charged particles to be 'pushed off' their straight line paths and into circular particle trajectories in the *xy*-plane (see Figure 2.7). (In the real detector where the particles travel in three dimensions, F is still perpendicular to both B and v, but now B and v are not always perpendicular - what happens in that case?)

By applying a magnetic field physicists can characterise charged particles from measurements of their radius of curvature and type of curvature; *i.e.* clockwise or anti-clockwise (see next section). This can be seen by studying equation 2.4: changing the sign of Q will also change the sign and therefore direction in which F acts; causing particles of opposite charge to curve in opposite directions. (Can you explain why the magnetic field along the z direction has no effect on the colliding electron and positron?)

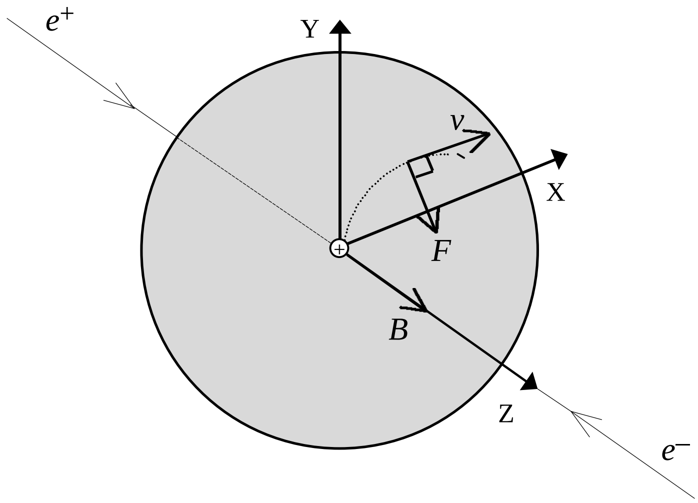

Figure 2.7 Showing the direction of the force F acting on a positive charge moving with velocity v in the *xy*-plane, under the influence of a magnetic field B. The dashed line shows the subsequent particle trajectory.

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