1001

$V = 2{,}5\ l = 2{,}5\ \text{dm}^3$

$h = 16{,}5\ \text{cm} = 1{,}65\ \text{dm}$

$V = \frac{1}{3} A_{\text{pohja}} \cdot h$

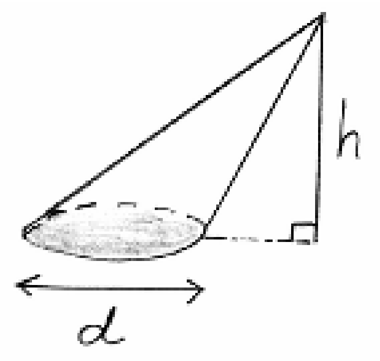
$V = \frac{1}{3}\pi r^2 \cdot h \qquad |\cdot 3$

$3V = \pi r^2 \cdot h$

$r^2 = \frac{3V}{\pi h}$

$r = _{(}\pm_{)}\sqrt{\frac{3V}{\pi h}} = \sqrt{\frac{3 \cdot 2{,}5\ \text{dm}^3}{\pi \cdot 1{,}65\ \text{dm}}} = 1{,}2028...\ \text{dm}$

$d = 2r = 2{,}4057...\ \text{dm} \approx 24\ \text{cm}$



Vastaus 24 cm

1002

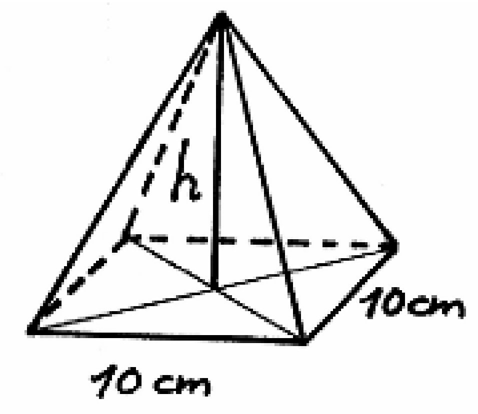
$V = 1\ l = 1\ \text{dm}^3 = 1000\ \text{cm}^3$

$V = \frac{1}{3} \cdot A_p h$

$1000 = \frac{1}{3} \cdot 10^2 \cdot h \qquad |\cdot 3$

$3000 = 100h \qquad |:100$

$h = \frac{3000}{100} = 30\ (\text{cm})$

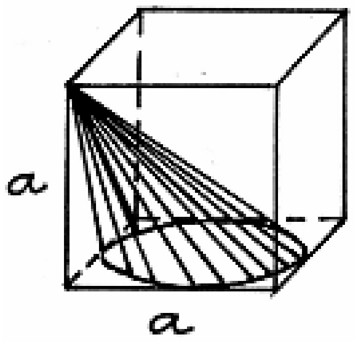


Vastaus 30 cm

1003

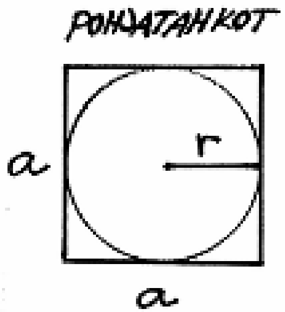
Kuution tilavuus

$$V_{\text{kuutio}} = a^3$$



Kartion tilavuus

$$\begin{aligned} V_{\text{kartio}} &= \frac{1}{3}A_p h \\ &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi\left(\frac{a}{2}\right)^2 a \\ &= \frac{\pi a^3}{12} \end{aligned}$$



$$\begin{aligned} \frac{V_{\text{kartio}}}{\text{V}_{\text{kuutio}}} \cdot 100\ \% &= \frac{\frac{\pi a^3}{12}}{a^3} \cdot 100\ \% \\ &= \frac{\pi a^3}{12} \cdot \frac{1}{a^3} \cdot 100\ \% \\ &= \frac{100\pi}{12}\ \% \\ &= \frac{25\pi}{3}\ \% \\ &\approx 26\ \% \end{aligned}$$

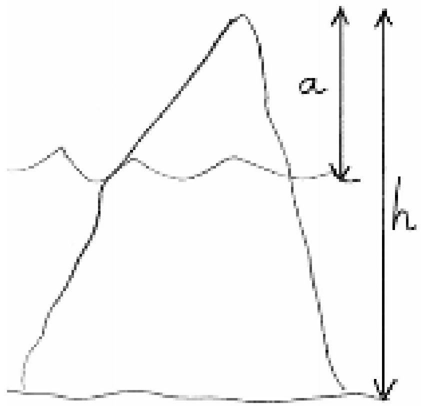
Vastaus $\frac{25\pi}{3}\ \% \approx 26\ \%$

1004

Merkitään kirjaimella *h* jäävuoren korkeutta ja kirjaimella *a* pinnan yläpuolelle jäävän osan korkeutta. Koska jäävuori on yhdenmuotoinen pinnan yläpuolelle jäävän osan kanssa, saadaan yhtälö

$$\frac{V_{\text{osa}}}{\text{V}_{\text{koko}}} = \left(\frac{a}{h}\right)^3 \qquad \left|\ \begin{aligned} V_{\text{osa}} &= 0{,}11 \cdot \text{V}_{\text{koko}} \\ a &= 10\,(\text{m}) \end{aligned}\right.$$

$$\begin{aligned} \frac{0{,}11 \cdot \text{V}_{\text{koko}}}{\text{V}_{\text{koko}}} &= \left(\frac{10}{h}\right)^3 \\ 0{,}11 &= \frac{1000}{h^3} \\ 0{,}11h^3 &= 1000 \\ h^3 &= \frac{1000}{0{,}11} \\ h &= \sqrt[3]{\frac{1000}{0{,}11}} \\ h &= 20{,}8706... \\ h &\approx 21\ (\text{m}) \end{aligned}$$

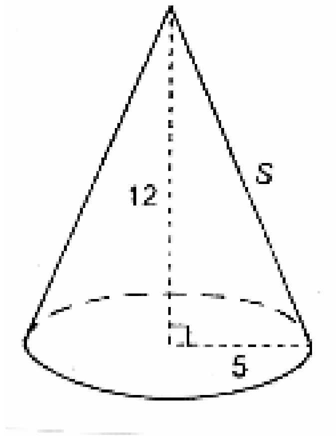


Vastaus Jäävuoren korkeus on 21 m.

1005

Tilavuus

$$V = \frac{1}{3} \cdot A_{\text{pohja}} \cdot h$$
$$= \frac{1}{3} \cdot \pi r^2 h$$
$$= \frac{1}{3} \pi \cdot 5^2 \cdot 12$$
$$= 100\pi$$



Sivujana saadaan Pythagoraan lauseella.

$$s^2 = 12^2 + 5^2$$
$$s = _{(}\pm_{)} \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$

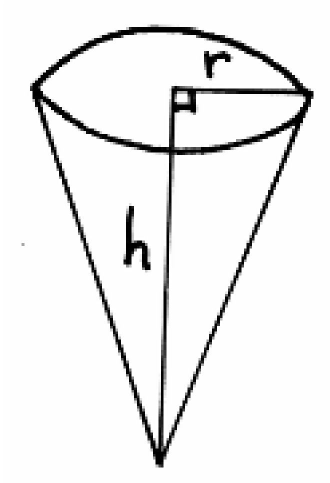
Vaipan ala

$$A = \pi r s = \pi \cdot 5 \cdot 13 = 65\pi$$

Vastaus tilavuus on 100π ja vaipan ala 65π

1006

Kannen säde



$$r = \frac{6{,}0 \text{ cm}}{2} = 3{,}0 \text{ cm}$$

Tuutin tilavuus

$$V = 1{,}2 \text{ dl} = 0{,}12 \; l = 0{,}12 \text{ dm}^3 = 120 \text{ cm}^3$$

Tuutin korkeus h saadaan yhtälöstä

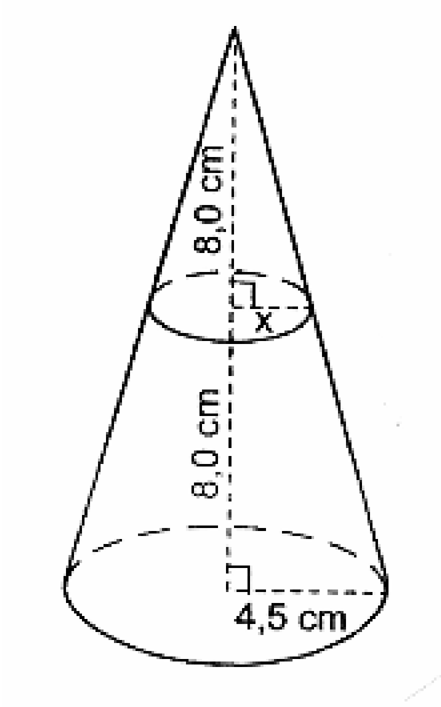
$$V = \frac{1}{3} \cdot A_{\text{pohja}} \cdot h$$
$$V = \frac{1}{3} \cdot \pi r^2 \cdot h \qquad \Big| \cdot 3$$
$$3V = \pi r^2 \cdot h \qquad \Big| : \pi r^2$$
$$h = \frac{3V}{\pi r^2}$$
$$h = \frac{3 \cdot 120}{\pi \cdot 3{,}0^2} = 12{,}732... \approx 13 \text{ (cm)}$$

Vastaus Tuutin korkeus on 13 cm.

1007

Pieni kartio ja iso kartio ovat yhdenmuotoiset, joten saadaan yhtälö

$$\frac{x}{4,5}=\frac{8}{16}$$
$$\frac{x}{4,5}=\frac{1}{2}$$
$$2x=4,5$$
$$x=\frac{4,5}{2}$$
$$x=2,25\ (\text{cm})$$



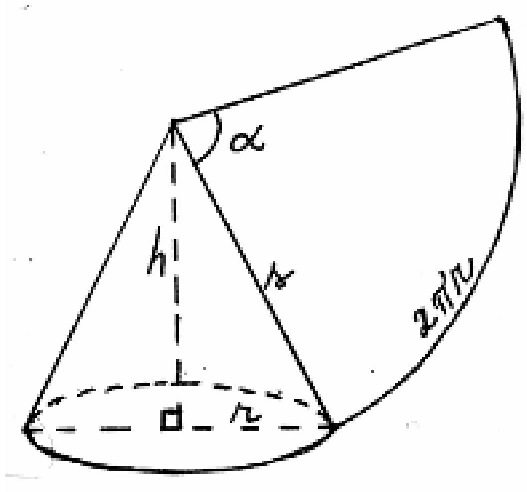
$$V=\frac{1}{3}\cdot\pi x^2\cdot 8$$
$$=\frac{1}{3}\cdot\pi\cdot(2,25)^2\cdot 8$$
$$=42,4115...$$
$$\approx 42\ (\text{cm}^3)$$

Vastaus 42 cm^3

1008

$r=6$ cm

$h=15$ cm



Sivujana s saadaan Pythagoraan lauseella.

$$s^2=h^2+r^2$$
$$s=_{(\pm)}\sqrt{15^2+6^2}=\sqrt{261}=3\sqrt{29}=16,15...\ (\text{cm})$$

a)

$$A_{\text{vaippa}}=\pi rs$$
$$=\pi\cdot 6\cdot 3\sqrt{29}$$
$$=18\pi\sqrt{29}$$
$$=304,52...$$
$$\approx 305\ (\text{cm}^2)$$

b) Ympyräsektorin kaari = pohjaympyrän kehä

$$\frac{\alpha}{360^\circ}\cdot 2\pi s = 2\pi r \quad \Big|:2\pi$$

$$\frac{\alpha}{360^\circ}\cdot s = r \quad \Big|\cdot 360^\circ$$

$$\alpha\cdot s = r\cdot 360^\circ \quad \Big|:s$$

$$\alpha = \frac{r\cdot 360^\circ}{s}$$

$$\alpha = \frac{6\cdot 360^\circ}{3\sqrt{29}}$$

$$\alpha = 133{,}7...^\circ \approx 134^\circ$$

b) **Tapa 2**

$$A_{\text{vaippa}} = \frac{\alpha}{360^\circ}\cdot \pi s^2 \quad \Big|\cdot 360^\circ$$

$$A_{\text{vaippa}}\cdot 360^\circ = \alpha\cdot \pi s^2 \quad \Big|:\pi s^2$$

$$\alpha = \frac{A_{\text{vaippa}}\cdot 360^\circ}{\pi s^2} \quad \Big|\, A_{\text{vaippa}} = 18\pi\sqrt{29}\ (\text{cm}^2),\ s = 3\sqrt{29}\ (\text{cm})$$

$$\alpha = \frac{18\pi\sqrt{29}\cdot 360^\circ}{\pi\left(3\sqrt{29}\right)^2}$$

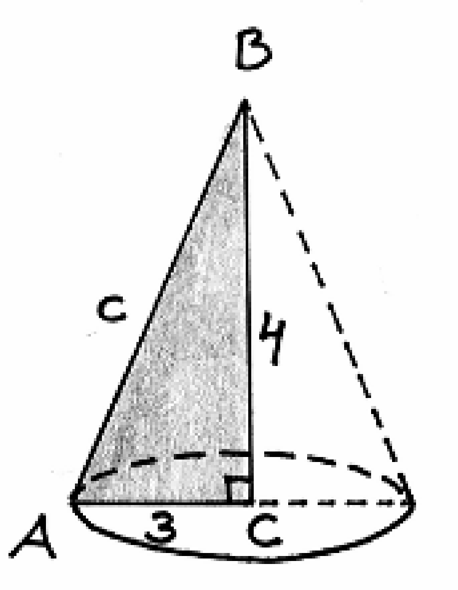
$$\alpha = \frac{2\cdot 360^\circ}{\sqrt{29}}$$

$$\alpha = 133{,}7...^\circ \approx 134^\circ$$

Vastaus a) $305\ \text{cm}^2$ b) 134°

1009

a)

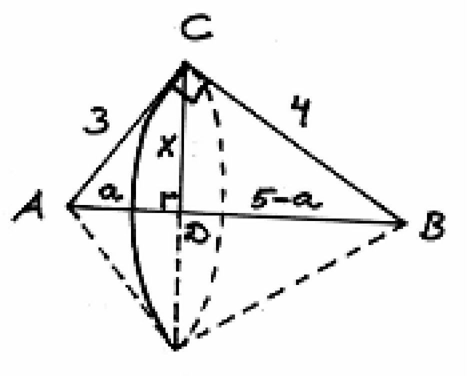


$$V = \frac{1}{3}A_p h$$

$$= \frac{1}{3}\cdot \pi\cdot 3^2\cdot 4$$

$$= 12\pi \approx 37{,}7$$

b) Hypotenuusan pituus



$$c^2 = 3^2 + 4^2$$

$$c^2 = 25$$

$$c = _{(}\pm_{)}\sqrt{25}$$

$$c = 5$$

$$V = \frac{1}{3}\pi x^2 a + \frac{1}{3}\pi x^2(5-a)$$

$$= \frac{1}{3}\pi x^2 a + \frac{5}{3}\pi x^2 - \frac{1}{3}\pi x^2 a$$

$$= \frac{5}{3}\pi x^2$$

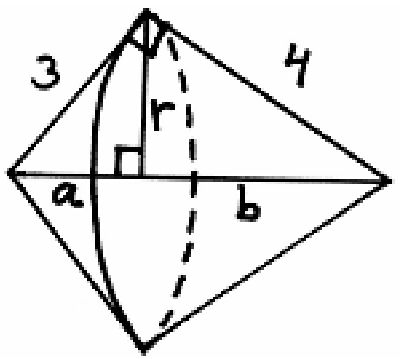
Ratkaistaan ensin *x*.

$$\frac{x}{4}=\frac{3}{5} \qquad \Big| \triangle ADX \sim \triangle ACB\ (kk)$$
$$5x=12$$
$$x=\frac{12}{5}$$

Tilavuus on

$$V=\frac{5}{3}\pi\cdot\left(\frac{12}{5}\right)^2=\frac{5\cdot\pi\cdot 12\cdot 12}{3\cdot 5\cdot 5}$$
$$=\frac{48\pi}{5}=9\frac{3}{5}\pi\approx 30,2$$

b) **Tapa 2**



Pythagoraan lauseen mukaan saadaan yhtälöryhmä

$$\begin{cases} (1)\ a^2+r^2=3^2 \\ (2)\ b^2+r^2=4^2 \\ (3)\ 3^2+4^2=(a+b)^2 \end{cases}$$

$$(3)\ (a+b)^2=25$$
$$a+b=_{(\pm)}\sqrt{25}$$
$$a+b=5$$
$$(4)\qquad a=5-b \ \Big|\text{sijoitetaan yhtälöön } (1)$$

$$\begin{cases} (5-b)^2+r^2=9 \\ b^2+r^2=16 \end{cases} \qquad \Big|\cdot(-1)$$

$$+\begin{cases} (5-b)^2+r^2=9 \\ -b^2-r^2=-16 \end{cases}$$

$$(5-b)^2-b^2=-7$$
$$25-10b+b^2-b^2=-7$$
$$-10b=-32$$
$$b=3,2 \qquad \Big|\text{Sijoitetaan yhtälöön } (4).$$
$$a=5-b=5-3,2=1,8$$

$$(1)\qquad a^2+r^2=9 \qquad \Big| a=1,8$$
$$1,8^2+r^2=9$$
$$r^2=5,76$$
$$r=_{(\pm)}\sqrt{5,76}$$
$$r=2,4$$

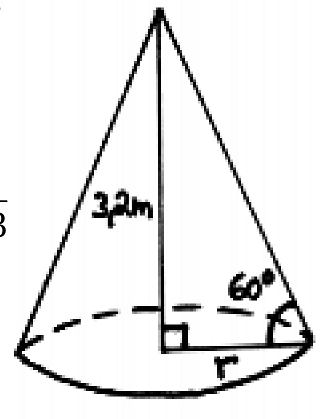
$$V = V_1 + V_2$$
$$= \frac{1}{3}\pi r^2 a + \frac{1}{3}\pi r^2 b$$
$$= \frac{1}{3}\pi r^2 (a+b) \qquad \left| \begin{array}{l} r = 2,4 \\ a = 1,8 \\ b = 3,2 \end{array} \right.$$
$$= \frac{1}{3}\pi \cdot 2,4^2 \cdot 5$$
$$= 9,6\pi$$
$$= 9\frac{6}{10}\pi$$
$$= 9\frac{3}{5}\pi = \frac{48\pi}{5} \approx 30,2$$

Vastaus a) $12\pi \approx 37,7$ b) $\frac{48\pi}{5} = 9\frac{3}{5}\pi \approx 30,2$

1010

a)

$$\tan 60^\circ = \frac{3,2}{r} \qquad |\cdot r$$
$$r \cdot \tan 60^\circ = 3,2 \qquad |: \tan 60^\circ$$
$$r = \frac{3,2}{\tan 60^\circ} \qquad |\tan 60^\circ = \sqrt{3}$$
$$r = \frac{3,2}{\sqrt{3}}$$
$$r = 1,847... \text{ (m)}$$



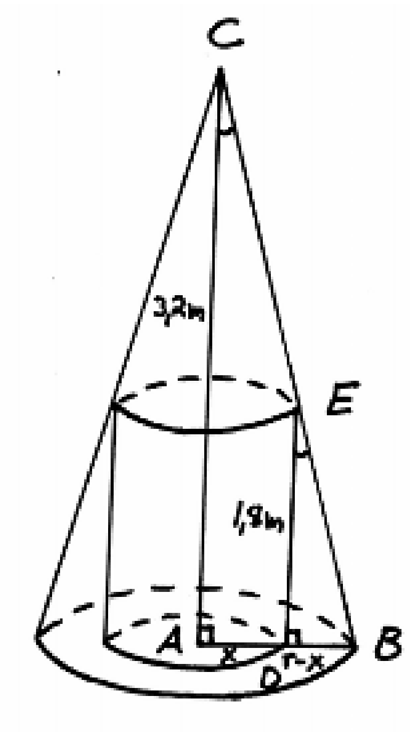
Kodan tilavuus

$$V = \frac{1}{3}\pi r^2 h \qquad \left| \begin{array}{l} r = 1,847... \text{ (m)} \\ h = 3,2 \text{ (m)} \end{array} \right.$$
$$= 11,43...$$
$$\approx 11 \text{ (m}^3\text{)}$$

b)

$$r=\frac{3,2}{\sqrt{3}}=1,847...\ (\text{m})$$

$$\triangle ABC \sim \triangle DBE \ (\text{kk})$$



$$\frac{3,2}{1,8}=\frac{r}{r-x}$$

$$3,2(r-x)=1,8r$$

$$3,2r-3,2x=1,8r$$

$$3,2r-1,8r=3,2x$$

$$1,4r=3,2x$$

$$x=\frac{1,4r}{3,2} \quad \Big| \, r=\frac{3,2}{\sqrt{3}}$$

$$x=\frac{1,4}{3,2}\cdot\frac{3,2}{\sqrt{3}}$$

$$x=\frac{1,4}{\sqrt{3}}=0,8082...\ (\text{m})$$

Alueen ala, jossa 180 cm pitkä henkilö voi seistä suorassa on

$$A=\pi x^2=\pi\cdot\left(\frac{1,4}{\sqrt{3}}\right)^2=2,052...\approx 2,1\ (\text{m}^2)$$

Vastaus a) 11 m^3 b) 2,1 m^2

1011

$$\triangle ABC \sim \triangle ADE \ (\text{kk})$$

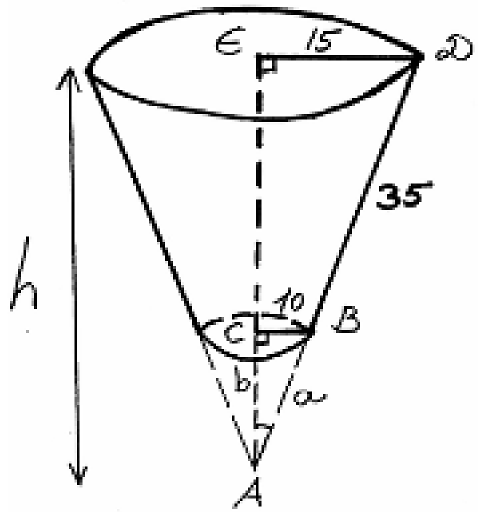
$$\frac{a}{a+35}=\frac{10}{15}$$

$$15a=10(a+35)$$

$$15a=10a+350$$

$$5a=350$$

$$a=70\ (\text{cm})$$



$$b^2+10^2=a^2$$

$$b^2+10^2=70^2$$

$$b^2=70^2-10^2$$

$$b=_{(\pm)}\sqrt{70^2-10^2}$$

$$b=\sqrt{4800}=69,282...\ (\text{cm})$$

$$h^2+15^2=105^2 \quad |\triangle AED$$

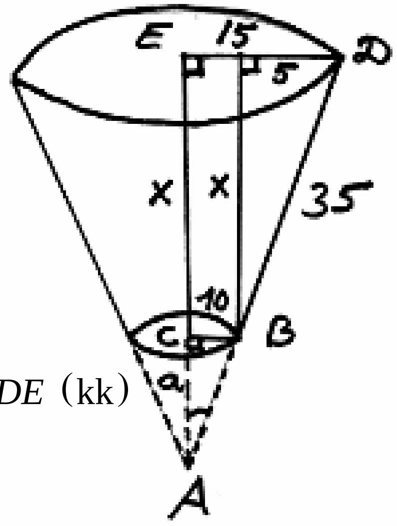
$$h^2=10800$$

$$h=_{(\pm)}\sqrt{10800}$$

$$V=\frac{1}{3}\pi\cdot 15^2h-\frac{1}{3}\pi\cdot 10^2\cdot b=17231,09...\ (\text{cm}^3)$$

Vastaus 17 litraa

Tapa 2



$$x^2+5^2=35^2$$

$$x^2=35^2-5^2$$

$$x=_{(}\pm_{)}\sqrt{35^2-5^2}$$

$$x=\sqrt{1200}\ \text{(cm)}$$

$$\frac{a}{a+x}=\frac{10}{15} \quad \left|\triangle ABC\sim\triangle ADE\ \text{(kk)}\right.$$

$$\frac{a}{a+\sqrt{1200}}=\frac{10}{15}$$

$$15a=10\left(a+\sqrt{1200}\right)$$

$$15a=10a+10\sqrt{1200}$$

$$5a=10\sqrt{1200}$$

$$a=2\sqrt{1200}\ \text{(cm)}$$

$$V_{\text{roskis}}=V_{\text{kartio}}^{\text{iso}}-V_{\text{kartio}}^{\text{pieni}}$$

$$=\frac{1}{3}\pi\cdot 15^2\cdot(a+x)-\frac{1}{3}\pi\cdot 10^2\cdot a \quad \left| \begin{array}{l} a=2\sqrt{1200} \\ x=\sqrt{1200} \end{array}\right.$$

$$=17\ 231{,}09...$$

$$\approx 17\ 000\ \left(\text{cm}^3\right)$$

Siis

$$V_{\text{roskis}}=17\ 000\ \text{cm}^3$$

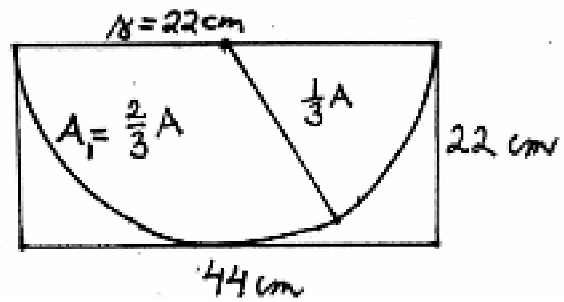
$$=17\ \text{dm}^3$$

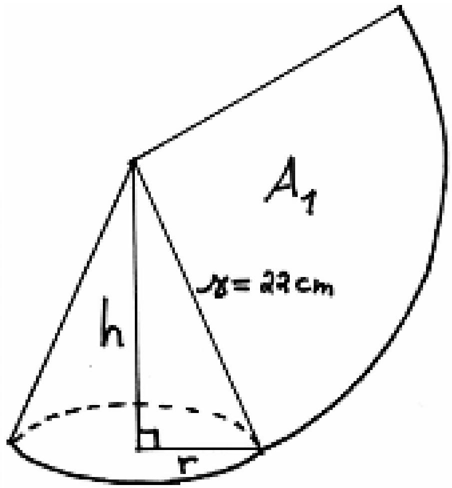
$$=17\ l$$

Vastaus 17 litraa

1012

Olkoon puoliympyrän ala *A*.





Kartion vaipan ala on

$$A_1 = \frac{2}{3}A = \frac{2}{3}\cdot\frac{1}{2}\pi s^2 = \frac{1}{3}\pi\cdot 22^2 = \frac{484\pi}{3}$$

Toisaalta kartion vaipan ala on

$$A_1 = \pi rs = \pi r\cdot 22 = 22\pi r$$

Saadaan yhtälö

$$22\pi r = \frac{484\pi}{3} \qquad |:22\pi$$

$$r = \frac{22}{3}$$

Pythagoraan lauseen mukaan

$$h^2 + r^2 = s^2$$

$$h^2 + \left(\frac{22}{3}\right)^2 = 22^2$$

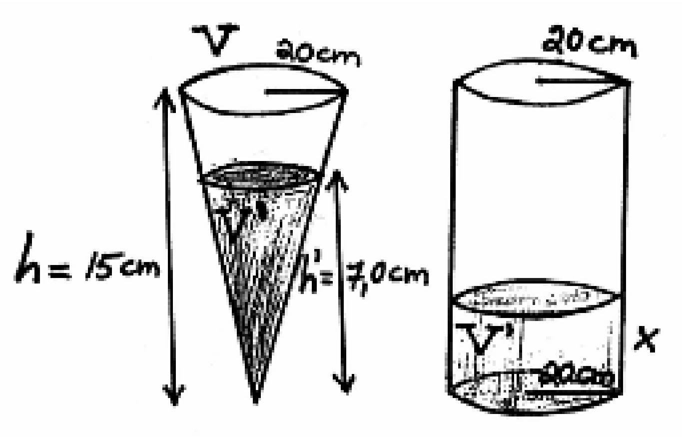
$$h^2 = 22^2 - \left(\frac{22}{3}\right)^2$$

$$h^2 = \frac{3872}{9}$$

$$h = {}_{(}\pm_{)}\sqrt{\frac{3872}{9}} = 20{,}74... \approx 21 \text{ (cm)}$$

Vastaus 21 cm

1013



Ympyräkartiot ovat yhdenmuotoisia, joten

$$\frac{V'}{V} = \left(\frac{h'}{h}\right)^3$$

$$\frac{\pi \cdot 20^2 \cdot x}{\frac{1}{3}\pi \cdot 20^2 \cdot 15} = \left(\frac{7}{15}\right)^3$$

$$\frac{x}{5} = \left(\frac{7}{15}\right)^3$$

$$x = \left(\frac{7}{15}\right)^3 \cdot 5$$

$$x = 0{,}5081... \text{ (cm)}$$

Vastaus 5 mm

1014

Kysytty korkeus on x cm.
Tilavuuksien suhteesta saadaan

$$\frac{V_1}{V_2 - V_1} = \frac{8}{117}$$

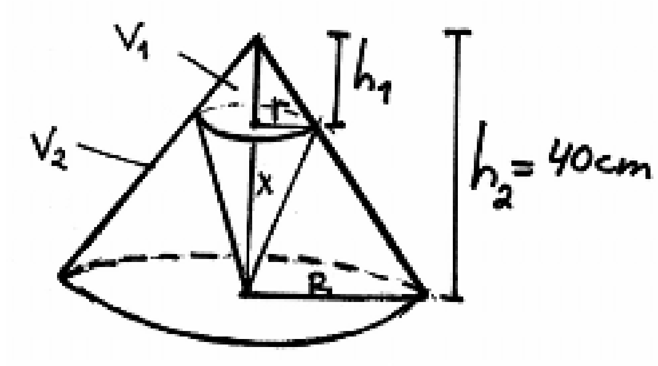
$$117V_1 = 8(V_2 - V_1)$$

$$117V_1 = 8V_2 - 8V_1$$

$$125V_1 = 8V_2$$

$$V_1 = \frac{8}{125}V_2$$

$$\frac{V_1}{V_2} = \frac{8}{125}$$



Toisaalta yhdenmuotoisten kartioiden tilavuuksien suhde on yhtä suuri kuin kartioiden korkeuksien suhteen kuutio. Siis

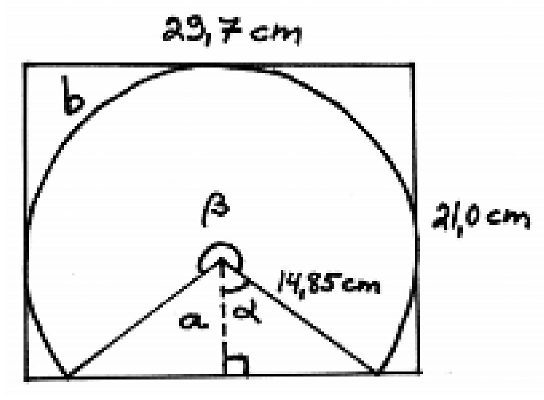
$$\frac{V_1}{V_2} = \left(\frac{40 - x}{40}\right)^3$$

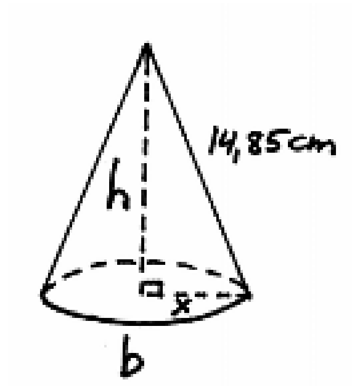
Saadaan yhtälö

$$\left(\frac{40-x}{40}\right)^3=\frac{8}{125}$$
$$\frac{40-x}{40}=\sqrt[3]{\frac{8}{125}}$$
$$\frac{40-x}{40}=\frac{2}{5}$$
$$5(40-x)=2\cdot 40$$
$$200-5x=80$$
$$-5x=-120$$
$$x=24\ (\text{cm})$$

Vastaus 24 cm

1015





$$a=21,0-14,85=6,15\ (\text{cm})$$

$$\cos\alpha=\frac{6,15}{14,85}$$
$$\alpha=65,534...^\circ$$

$$\beta=360^\circ-2\alpha=228,93...^\circ$$

$$b=\frac{\beta}{360^\circ}\cdot 2\pi\cdot 14,85=59,334...\ (\text{cm})$$

Saadaan yhtälö

$$2\pi x = b$$

$$x = \frac{b}{2\pi} = 9,4433...\ (\text{cm})$$

Pythagoraan lauseen mukaan

$$h^2 + x^2 = 14,85^2$$

$$h^2 = 14,85^2 - x^2$$

$$h = _{(\pm)}\sqrt{14,85^2 - x^2}$$

$$h = 11,460...\ (\text{cm})$$

Kartion tilavuus on

$$V = \frac{1}{3}A_p h = \frac{1}{3}\cdot \pi x^2 h = 1070,2...\ (\text{cm}^3)$$

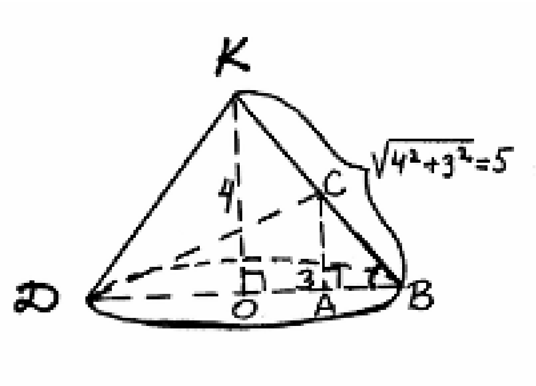
Siis

$$V = 1070,2...\ \text{cm}^3 = 1,0702...\ \text{dm}^3 \approx 1,1\ l$$

Vastaus 1,1 litraa

1016

Sivujanan keskipisteestä kauimpana oleva pohjaympyrän piste on vastakkaisella puolella kartiota.



a) Kolmiot *OBK* ja *ABC* ovat yhdenmuotoiset (kk).

Koska $BC = \frac{1}{2}BK = \frac{5}{2}$, on $AB = \frac{1}{2}OB = \frac{3}{2}$ ja $AC = \frac{1}{2}OK = 2$, joten kolmiosta DAC saadaan

$$DC = \sqrt{DA^2 + AC^2}$$

$$= \sqrt{\left(3+\frac{3}{2}\right)^2 + 2^2} = \sqrt{\left(\frac{9}{2}\right)^2 + 4} = \sqrt{\frac{81+16}{4}}$$

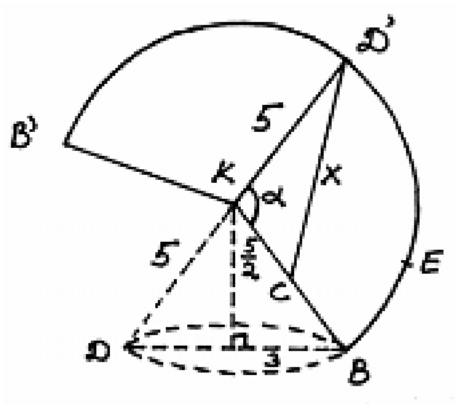
$$= \frac{\sqrt{97}}{2} = 4,924... \approx 4,9$$

b) Kartion pohjan piiri on

$$p = 2\pi r = 2\pi \cdot 3 = 6\pi$$

Tällöin kaaren BED' pituus on

$$b = \frac{p}{2} = \frac{6\pi}{2} = 3\pi$$



Toisaalta

$$b=\frac{\alpha}{360^\circ}\cdot 2\pi r=\frac{\alpha}{360^\circ}\cdot 2\pi\cdot 5=\frac{10\pi\alpha}{360^\circ}=\frac{\pi\alpha}{36^\circ}$$

Siis

$$\frac{\pi\alpha}{36^\circ}=3\pi$$
$$\alpha=108^\circ$$

Kosinilauseen mukaan

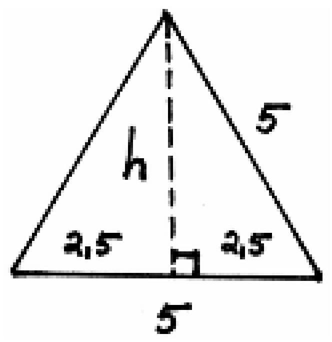
$$x^2=\left(\frac{5}{2}\right)^2+5^2-2\cdot\frac{5}{2}\cdot 5\cdot\cos 108^\circ$$
$$x^2=\frac{25}{4}+{}^{4)}25-{}^{4)}25\cos 108^\circ$$
$$x^2=\frac{25+100-100\cos 108^\circ}{4}$$
$$x^2=\frac{125-100\cos 108^\circ}{4}$$
$$x=_{(\pm)}\sqrt{\frac{25\cdot(5-4\cos 108^\circ)}{4}}$$
$$x=\frac{5\sqrt{5-4\cos 108^\circ}}{2}\approx 6,2$$

Vastaus a) $\frac{\sqrt{97}}{2}\approx 4,9$ b) $\frac{5}{2}\sqrt{5-4\cos 108^\circ}\approx 6,2$

1017

Tahkot ovat yhteneviä tasasivuisia kolmioita.
Kolmion korkeus saadaan Pythagoraan lauseella.

$$h^2+2,5^2=5,0^2$$
$$h^2=5,0^2-2,5^2$$
$$h=_{(\pm)}\sqrt{5^2-2,5^2}$$
$$h=\sqrt{18,75}$$
$$h=4,3301...\ (\text{cm})$$

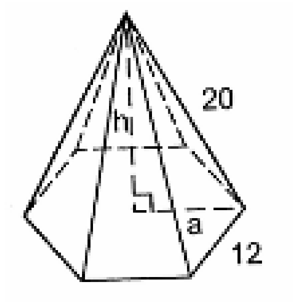


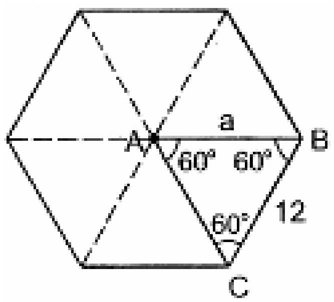
Teepussien pinta-ala on

$$A=4\cdot\frac{5\cdot\sqrt{18,75}}{2}$$
$$=43,3012...$$
$$\approx 43\ (\text{cm}^2)$$

Vastaus 43 cm^2

1018





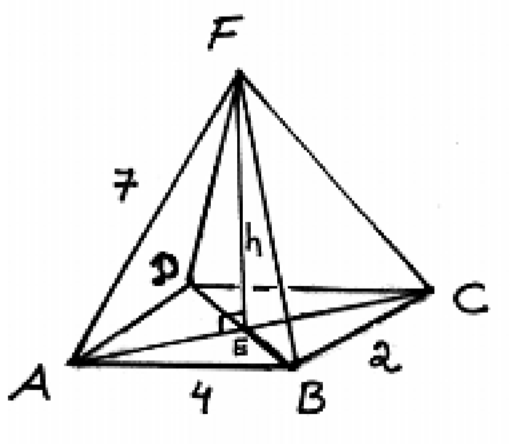
Kolmio ABC on tasasivuinen, joten $a = 12$.
Korkeus h saadaan Pythagoraan lauseella.

$$h^2 + a^2 = 20^2$$
$$h^2 + 12^2 = 20^2$$
$$h^2 = 20^2 - 12^2$$
$$h = _{(}\pm_{)}\sqrt{20^2 - 12^2}$$
$$h = \sqrt{256}$$
$$h = 16$$

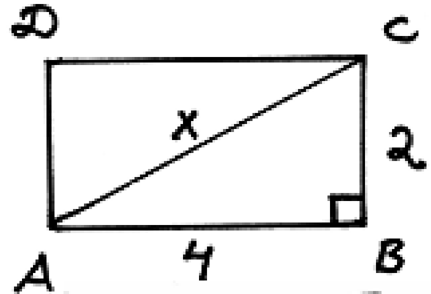
Vastaus 16

1019

a) Suora pyramidi



Pohja $ABCD$:

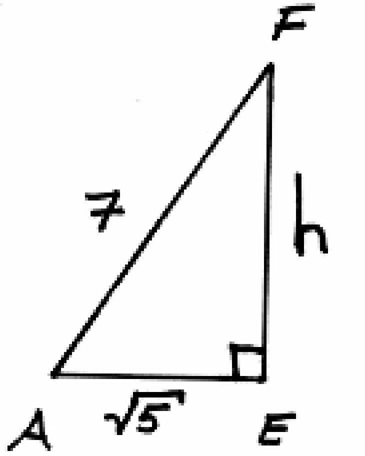


$$x^2 = 4^2 + 2^2$$
$$x^2 = 20$$
$$x = _{(}\pm_{)}\sqrt{20}$$
$$x = \sqrt{4 \cdot 5}$$
$$x = 2\sqrt{5}$$

Siis

$$AE = \frac{x}{2} = \sqrt{5}\ (\text{cm})$$

Kolmio AEF:



$$h^2 = 7^2 - \left(\sqrt{5}\right)^2$$
$$h^2 = 49 - 5$$
$$h^2 = 44$$
$$h = _{(}\pm_{)}\sqrt{44}$$
$$h = 2\sqrt{11}\ (\text{cm})$$

Tilavuus

$$V = \frac{1}{3}A_p h = \frac{1}{3} \cdot 4 \cdot 2 \cdot 2\sqrt{11} = \frac{16}{3}\sqrt{11} = 17{,}688... \approx 18\ (\text{cm}^3)$$

b) Vino pyramidi

$$V = \frac{1}{3}Ah \qquad \left| \begin{array}{l} A = 8 \ \left(\text{cm}^3\right) \\ h = 7 \ \left(\text{cm}\right) \end{array} \right.$$

$$= \frac{1}{3} \cdot 8 \cdot 7 = \frac{56}{3} = 18,66... \approx 19 \ \left(\text{cm}^3\right)$$

Vastaus a) 18 cm^3 b) 19 cm^3

1020

Yhdenmuotoisten kappaleiden tilavuuksien suhde on yhtä suuri kuin mittakaavan kuutio. Siis

Yhdenmuotoisten kappaleiden pinta-alojen suhde on yhtä suuri kuin mittakaavan neliö. Siis

$$\frac{V'}{V} = k^3$$

$$\frac{8}{27} = k^3$$

$$k = \sqrt[3]{\frac{8}{27}}$$

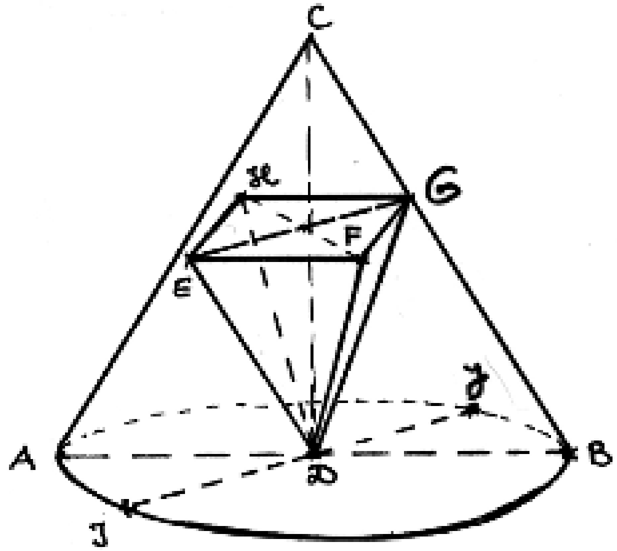
$$k = \frac{2}{3}$$

$$\frac{A'}{A} = k^2$$

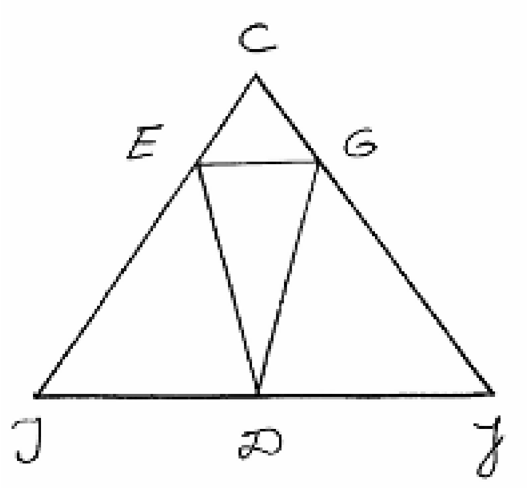
$$\frac{A'}{A} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Vastaus 4:9

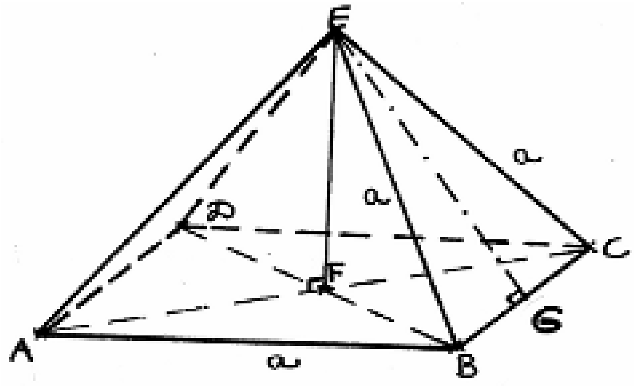
1021



Tasoleikkaus:



1022



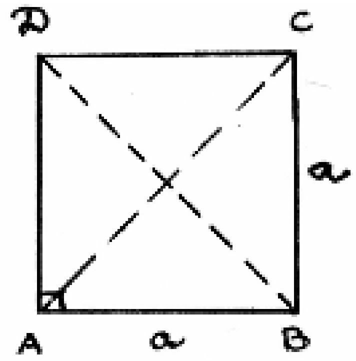
Pyramidin korkeus EF

$$AC^2 = a^2 + a^2$$

$$AC^2 = 2a^2$$

$$AC = _{(\pm)}\sqrt{2a^2}$$

$$AC = a\sqrt{2}$$



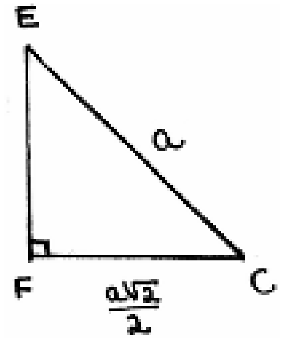
$$FC = \frac{AC}{2} = \frac{a\sqrt{2}}{2}$$

$$EF^2 = a^2 - \left(\frac{a\sqrt{2}}{2}\right)^2$$

$$EF^2 = a^2 - \frac{2a^2}{4}$$

$$EF^2 = \frac{a^2}{2}$$

$$EF = \frac{a}{\sqrt{2}}$$



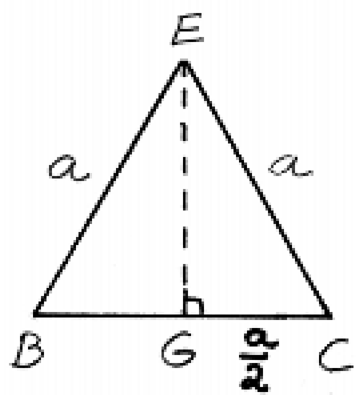
Sivutahkon korkeus EG

$$EG^2 = a^2 - \left(\frac{a}{2}\right)^2$$

$$EG^2 = a^2 - \frac{a^2}{4}$$

$$EG^2 = \frac{3a^2}{4}$$

$$EG = \frac{a\sqrt{3}}{2}$$



Vaipan ala on

$$A_{\text{vaippa}} = 4A_{BCE} = 4 \cdot \frac{1}{2} \cdot a \cdot \frac{a\sqrt{3}}{2} = a^2\sqrt{3}$$

Pohjan ala on

$$A_{\text{pohja}} = A_{ABCD} = a \cdot a = a^2$$

Kokonaispinta-ala on

$$A = A_{\text{vaippa}} + A_{\text{pohja}} = a^2\sqrt{3} + a^2 = a^2\left(\sqrt{3}+1\right) \approx 2{,}7a^2$$

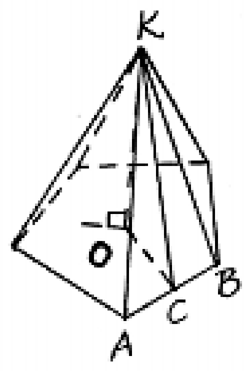
Tilavuus on

$$V = \frac{1}{3}A_{\text{pohja}} \cdot h = \frac{1}{3} \cdot a^2 \cdot \frac{a}{\sqrt{2}} \overset{\sqrt{2})}{=} \frac{a^3}{3\sqrt{2}} = \frac{a^3\sqrt{2}}{6} \approx 0{,}24a^3$$

Vastaus Kokonaispinta-ala on $a^2\left(\sqrt{3}+1\right) \approx 2{,}7a^2$

Tilavuus on $\dfrac{a^3}{3\sqrt{2}} = \dfrac{a^3\sqrt{2}}{6} \approx 0{,}24a^3$

1023



$$\alpha = \frac{360^\circ}{5} = 72^\circ$$

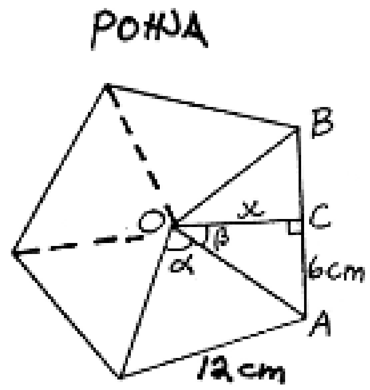
$$\beta = \frac{\alpha}{2} = 36^\circ$$

$$\tan\beta = \frac{6}{x}$$

$$\tan 36^\circ = \frac{6}{x}$$

$$x \cdot \tan 36^\circ = 6$$

$$x = \frac{6}{\tan 36^\circ}$$



Tilavuus:

$$V = \frac{1}{3} A_{\text{pohja}} \cdot h$$

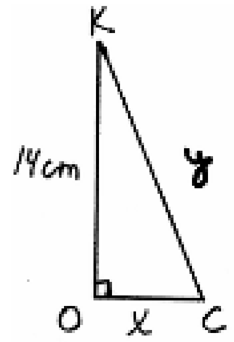
$$= \frac{1}{3}\left(5 \cdot \frac{12 \cdot x}{2}\right) \cdot 14$$

$$= \frac{1}{3} \cdot 30 \cdot \frac{6}{\tan 36^\circ} \cdot 14$$

$$= \frac{840}{\tan 36^\circ} = 1156{,}16\ldots \approx 1200 \ (\text{cm}^3)$$

$$V \approx 1200 \ \text{cm}^3 = 1{,}2 \ \text{dm}^3$$

Vaipan ala:

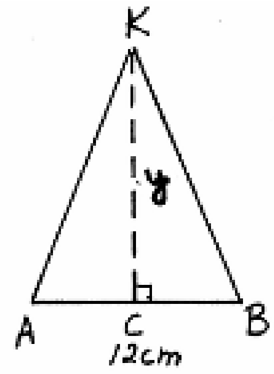


$$y^2 = 14^2 + x^2$$

$$y^2 = 14^2 + \left(\frac{6}{\tan 36^\circ}\right)^2$$

$$y = _{(}\pm_{)} \sqrt{264{,}19937\ldots}$$

$$y = 16{,}254211\ldots \ (\text{cm})$$



$$A_{\text{vaippa}} = 5 A_{ABK}$$

$$= 5 \cdot \frac{12 \cdot y}{2}$$

$$= 487{,}626\ldots \approx 490 \ (\text{cm}^2)$$

$$A_{\text{vaippa}} \approx 490 \ \text{cm}^2 = 4{,}9 \ \text{dm}^2$$

Vastaus Tilavuus $1{,}2 \ \text{dm}^3$, vaipan ala $4{,}9 \ \text{dm}^2$

1024

$t_1 = 30$ a $\quad h_1 = 147$ m
$t_2 = ?$ $\quad h_2 = 144$ m

Tilavuuksien suhde on mittakaavan kuutio. Siis

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

Toisaalta tilavuus V ja rakentamisaika t ovat suoraan verrannolliset, joten

$$V = at, \text{ vakio } a \neq 0$$

Siis

$$\frac{V_1}{V_2} = \frac{at_1}{at_2} = \frac{t_1}{t_2}$$

Saadaan yhtälö

$$\frac{t_1}{t_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{t_1}{t_2} = \frac{h_1^3}{h_2^3}$$

$$t_2 = \frac{t_1 \cdot h_2^3}{h_1^3} = \frac{30 \cdot 144^3}{147^3} = 28{,}200... \approx 28 \text{ (a)}$$

Vastaavasti

$t_1 = 30$ a $\quad h_1 = 147$ m
$t_3 = ?$ $\quad h_3 = 66$ m

$$\frac{t_1}{t_3} = \left(\frac{h_1}{h_3}\right)^3$$

$$\frac{t_1}{t_3} = \frac{h_1^3}{h_3^3}$$

$$t_3 = \frac{t_1 \cdot h_3^3}{h_1^3} = \frac{30 \cdot 66^3}{147^3} = 2{,}715... \approx 3 \text{ (a)}$$

Vastaus Rakennusajat olivat 28 a ja 3 a.

1025

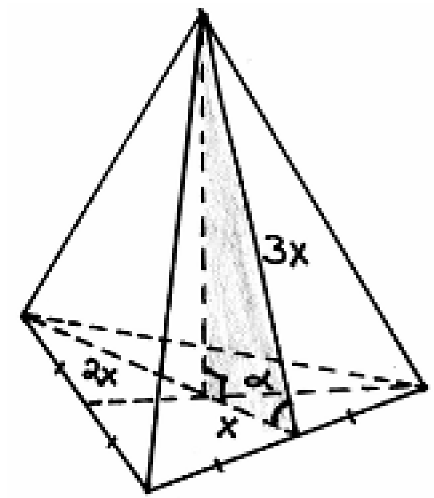
a) Kaikki tahkot ovat yhteneviä tasasivuisia kolmioita.
Mediaanit jakavat toisensa 2:1 kärjestä lukien.
Olkoon mediaanien pituus *3x*.
Pohjatahkon keskipiste on mediaanien leikkauspiste.

$$\cos\alpha = \frac{x}{3x}$$

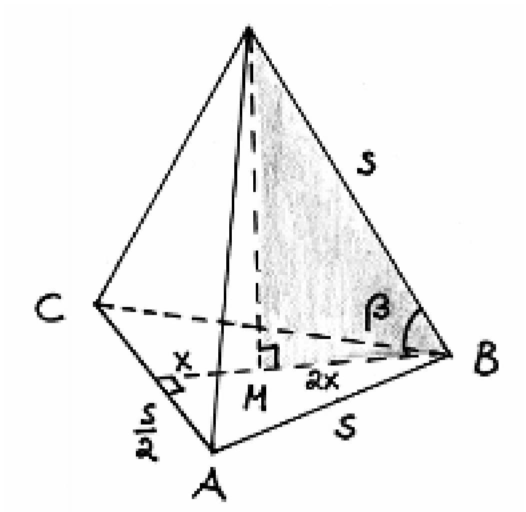
$$\cos\alpha = \frac{1}{3}$$

$$\alpha = 70{,}528...^\circ$$

$$\alpha \approx 70{,}5^\circ$$



b) Olkoon sivusärmän pituus *s* ja mediaanin pituus *3x*. Olkoon *M* mediaanien leikkauspiste.



Pohjatahko (tasasivuinen kolmio):

$$(3x)^2 + \left(\frac{s}{2}\right)^2 = s^2$$

$$9x^2 + \frac{s^2}{4} = s^2$$

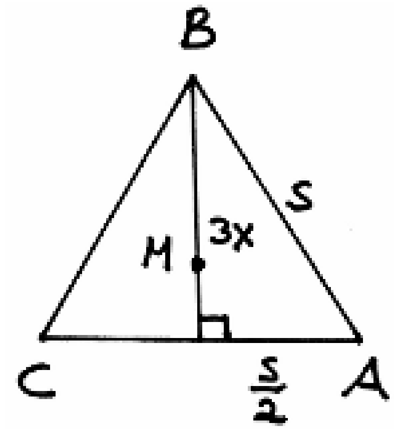
$$9x^2 = s^2 - \frac{s^2}{4}$$

$$9x^2 = \frac{3s^2}{4} \qquad \left| \cdot \frac{1}{9}\right.$$

$$x^2 = \frac{3s^2}{4} \cdot \frac{1}{9}$$

$$x^2 = \frac{s^2}{12}$$

$$x = _{(}\pm_{)} \frac{s}{\sqrt{12}}$$



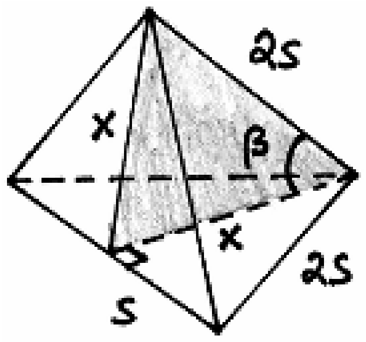
$$\cos\beta = \frac{2x}{s} = \frac{\frac{2s}{\sqrt{12}}}{s} = \frac{2s}{\sqrt{12}} \cdot \frac{1}{s} = \frac{2}{\sqrt{12}}$$

$$\beta = 54{,}735...^\circ$$

$$\beta \approx 54{,}7^\circ$$

b) **Tapa 2**

Pythagoraan lauseen mukaan



$$x^2 + s^2 = (2s)^2$$
$$x^2 = 3s^2$$
$$x = s\sqrt{3}$$

Kosinilauseen mukaan

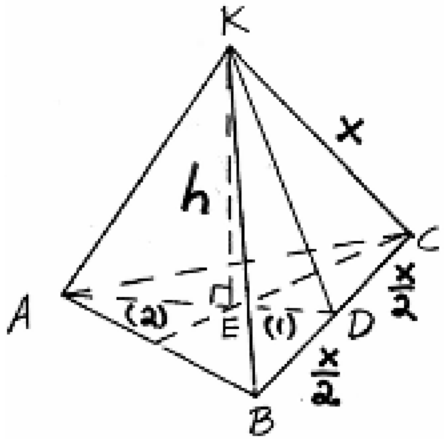
$$x^2 = x^2 + (2s)^2 - 2\cdot x\cdot 2s\cdot \cos\beta$$
$$4xs\cos\beta = 4s^2$$
$$\cos\beta = \frac{4s^2}{4xs}$$
$$\cos\beta = \frac{s}{x} \qquad \Big| x = s\sqrt{3}$$
$$\cos\beta = \frac{1}{\sqrt{3}}$$
$$\beta = 54{,}735...^\circ$$
$$\beta \approx 54{,}7^\circ$$

Vastaus a) $70{,}5^\circ$ b) $54{,}7^\circ$

1026

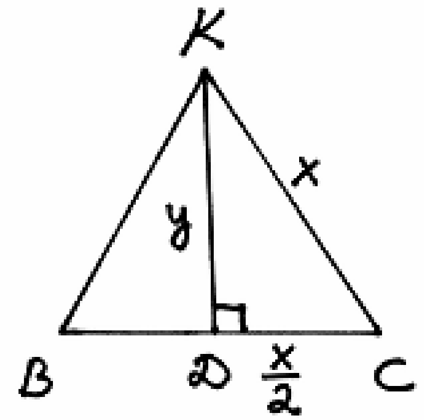
Tilavuus $V = 2{,}0\ \text{dl} = 0{,}2\ l = 0{,}2\ \text{dm}^3 = 200\ \text{cm}^3$

Säännöllisen tetraedrin kaikki tahkot ovat yhteneviä tasasivuisia kolmioita. Olkoon tetraedrin särmä x ja korkeus h. Käytetään pituusyksikkönä senttimetriä.



Säännöllisen tetraedrin korkeusjana yhdistää tetraedrin kärjen ja vastakkaisen tahkon keskipisteen (mediaanien leikkauspisteen).

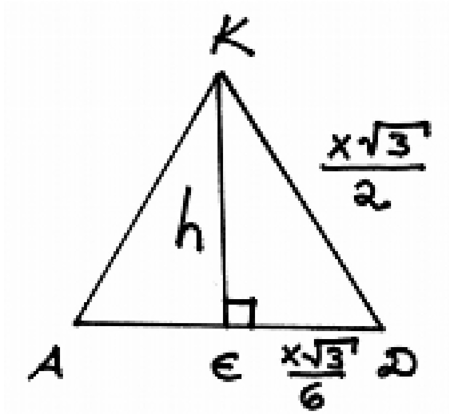
Pythagoraan lauseen mukaan



$$KD^2 + \left(\frac{x}{2}\right)^2 = x^2$$
$$KD^2 + \frac{x^2}{4} = x^2$$
$$KD^2 = \frac{3x^2}{4}$$
$$KD = \frac{x\sqrt{3}}{2}\ (\text{cm})$$

$$AD = KD = \frac{x\sqrt{3}}{2}\ (\text{cm})$$

$$ED = \frac{1}{3}AD = \frac{x\sqrt{3}}{6}\ (\text{cm})$$



$$h^2 + ED^2 = KD^2$$

$$h^2 + \left(\frac{x\sqrt{3}}{6}\right)^2 = \left(\frac{x\sqrt{3}}{2}\right)^2$$

$$h^2 = {}^{3)}\frac{3x^2}{4} - \frac{x^2}{12}$$

$$h^2 = \frac{8x^2}{12}^{2} = \frac{2x^2}{3}$$

$$h = \frac{x\sqrt{2}}{\sqrt{3}}\ (\text{cm})$$

Pyramidin tilavuus on

$$V = \frac{1}{3}A_p h = \frac{1}{3}A_{ABC}h = \frac{1}{3}A_{BCK}h$$

$$= \frac{1}{3}\cdot\frac{BC\cdot KD}{2}\cdot h$$

$$= \frac{1}{6}\cdot x\cdot\frac{x\sqrt{3}}{2}\cdot\frac{x\sqrt{2}}{\sqrt{3}}$$

$$= \frac{x^3\sqrt{2}}{12}\ (\text{cm}^3)$$

Saadaan yhtälö

$$\frac{x^3\sqrt{2}}{12} = 200 \qquad |\cdot 12$$

$$x^3\sqrt{2} = 2400$$

$$x^3 = \frac{2400}{\sqrt{2}}$$

$$x = \sqrt[3]{\frac{2400}{\sqrt{2}}}$$

$$x = 11{,}927...$$

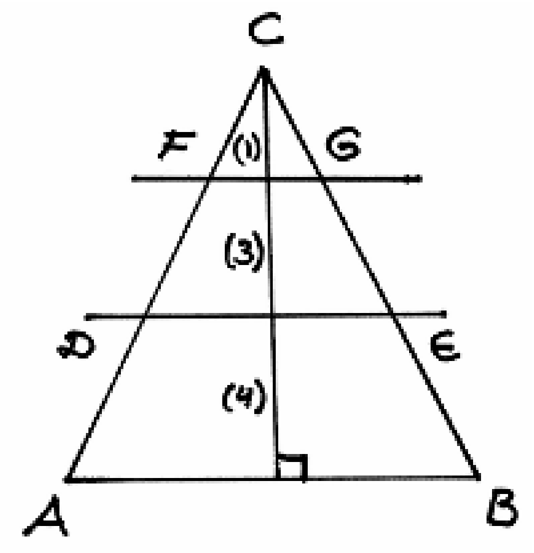
$$x \approx 12\ (\text{cm})$$

Vastaus 12 cm

1027

Leikataan pyramidi huipun *C* kautta kohtisuorasti pohjaa vastaan. Saadaan tasoleikkaus.

$\triangle FGC \sim \triangle ABC$ (kk)



Mittakaava on $k = \frac{1}{8}$, joten

$$\frac{V_{FGC}}{V_{ABC}} = k^3 = \left(\frac{1}{8}\right)^3 = \frac{1}{512}$$

Siis $V_{\text{ylin}} = V_{FGC} = \frac{1}{512} \cdot V_{ABC}$

$\triangle DEC \sim \triangle ABC$ (kk)

Mittakaava on $k = \frac{4}{8} = \frac{1}{2}$, joten

$$\frac{V_{DEC}}{V_{ABC}} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Siis $V_{DEC} = \frac{1}{8} V_{ABC}$ ja $V_{\text{alin}} = V_{ABC} - V_{DEC} = \frac{7}{8} V_{ABC}$

Suhde on

$$\frac{V_{\text{ylin}}}{V_{\text{alin}}} = \frac{\frac{1}{512} V_{ABC}}{\frac{7}{8} V_{ABC}} = \frac{1}{512} : \frac{7}{8} = \frac{1}{512} \cdot \frac{8}{7} = \frac{1}{448}$$

Vastaus Suhde on 1:448