1101

$d = 12,6\ \text{cm}$
$r = 6,3\ \text{cm}$

a) Pinta-ala on

$$A = 4\pi r^2 = 4\pi(6,3\ \text{cm})^2 = 498,75...\ \text{cm}^2 \approx 499\ \text{cm}^2$$

b) Tilavuus on

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot (6,3\ \text{cm})^3 = 1047,39...\ \text{cm}^3$$
$$\approx 1050\ \text{cm}^3 = 1,05\ \text{dm}^3 = 1,05\ l$$

Vastaus a) $499\ \text{cm}^2$ b) 1,05 litraa

1102

a) Tilavuus on

$$V = \frac{4}{3}\pi r^3 \quad |\cdot 3$$
$$3V = 4\pi r^3 \quad |:4\pi$$
$$\frac{3V}{4\pi} = r^3$$
$$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3 \cdot 10\ \text{m}^3}{4\pi}} = 1,3365...\ \text{m}$$

$$d = 2r = 2,6730...\ \text{m} \approx 2,7\ \text{m}$$

b) Ala on

$$A = 4\pi r^2 = 4\pi \cdot (1,3365...\ \text{m})^2 = 22,446...\ \text{m}^2 \approx 22\ \text{m}^2$$

Vastaus a) 2,7 m b) $22\ \text{m}^2$

1103

$$p_1 = 68 \ (\text{cm}) \qquad p_2 = 71 \ (\text{cm})$$
$$2\pi r_1 = 68 \qquad 2\pi r_2 = 71$$
$$r_1 = \frac{68}{2\pi} \qquad r_2 = \frac{71}{2\pi}$$

$$\begin{aligned} V_1 &= \frac{4}{3}\pi r_1^3 \\ &= \frac{4}{3}\pi\left(\frac{68}{2\pi}\right)^3 \\ &= 5309{,}77... \\ &\approx 5300 \ (\text{cm}^3) \end{aligned} \qquad \begin{aligned} V_1 &= \frac{4}{3}\pi r_2^3 \\ &= \frac{4}{3}\pi\left(\frac{71}{2\pi}\right)^3 \\ &= 6043{,}99... \\ &\approx 6000 \ (\text{cm}^3) \end{aligned}$$

Vastaus 5,3–6,0 litraa

1104

Vesimelonin säde on

$$r_1 = \frac{28{,}6 \ \text{cm}}{2} = 14{,}3 \ \text{cm}$$

Vesimelonin sisäosan säde on

$$r_2 = 14{,}3 \ \text{cm} - 2{,}2 \ \text{cm} = 12{,}1 \ \text{cm}$$

Vesimelonin tilavuus on

$$V_{\text{koko}} = \frac{4}{3}\pi r_1^3$$

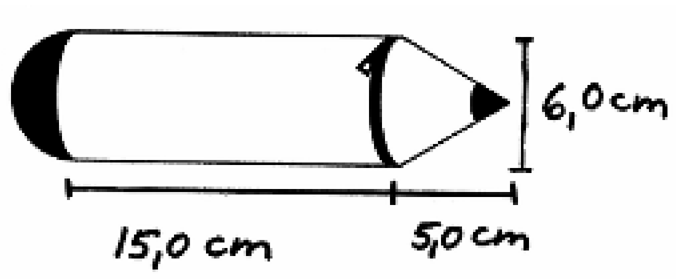
Syötävän osan tilavuus on

$$V_{\text{sisus}} = \frac{4}{3}\pi r_2^3$$

Syötävän osan tilavuus melonin tilavuudesta prosentteina on

$$\begin{aligned} \frac{V_{\text{sisus}}}{V_{\text{koko}}} &= \frac{\frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi r_1^3} \cdot 100 \ \% \\ &= \left(\frac{r_2}{r_1}\right)^3 \cdot 100 \ \% \\ &= \left(\frac{12{,}1}{14{,}3}\right)^3 \cdot 100 \ \% \\ &= 60{,}582... \ \% \\ &\approx 61 \ \% \end{aligned}$$

1105

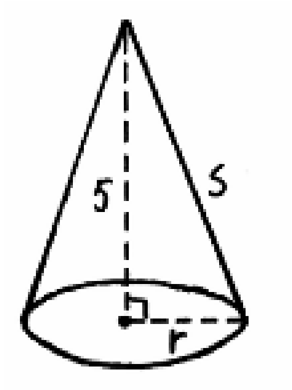


a) Lasketaan kotelon tilavuus

$$V = V_{\text{kartio}} + V_{\text{lieriö}} + V_{\text{puolipallo}}$$
$$= \frac{1}{3}\pi r^2 \cdot 5 + \pi r^2 \cdot 15 + \frac{1}{2}\cdot\frac{4}{3}\pi r^3$$
$$= \pi r^2\left(\frac{5}{3} + 15 + \frac{2}{3}r\right) \qquad | r = 3{,}0 \text{ (cm)}$$
$$= 527{,}78...\ (\text{cm}^3)$$
$$V \approx 530 \text{ cm}^3 = 0{,}53 \text{ dm}^3 = 0{,}53\ l$$

b) Lasketaan kotelon pinta-ala

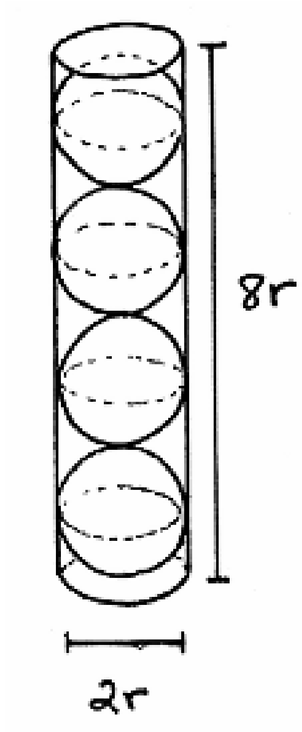
$$A = A_{\text{kartio}} + A_{\text{lieriö}} + A_{\text{puolipallo}}$$
$$= \pi r s + 2\pi r \cdot 15 + \frac{1}{2}\cdot 4\pi r^2$$
$$\pi r(s + 30 + 2r) \qquad s = \sqrt{5^2 + r^2}$$
$$= 394{,}24... \qquad = \sqrt{25+9}$$
$$\approx 390\ (\text{cm}^2) \qquad = \sqrt{34}\ (\text{cm})$$



Vastaus a) 0,53 litraa b) 390 cm^2

1106

pallon säde r
pallojen tilavuus



$$V_{\text{pallot}} = 4\cdot\frac{4}{3}\pi r^3 = \frac{16\pi r^3}{3}$$

lieriön tilavuus $V_{\text{lieriö}} = \pi r^2 \cdot 8r = 8\pi r^3$

tyhjä osuus

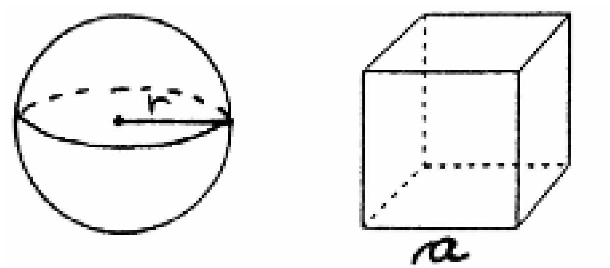
$$V_{\text{lieriö}} - V_{\text{pallot}} = 8\pi r^3 - \frac{16\pi r^3}{3}$$
$$= \frac{24\pi r^3 - 16\pi r^3}{3}$$
$$= \frac{8\pi r^3}{3}$$

Lasketaan tyhjän tilan osuus pakkauksesta.

$$\frac{V_{\text{lieriö}} - V_{\text{pallot}}}{V_{\text{lieriö}}}\cdot 100\ \% = \frac{8\pi r^3}{3\cdot 8\pi r^3}\cdot 100\ \% = 33\frac{1}{3}\ \% \approx 33\ \%$$

Vastaus 33 %

1107



$$m_{\text{pallo}} = m_{\text{kuutio}} \qquad \left| \rho = \frac{m}{V} \quad \text{eli} \quad m = \rho V \right.$$

$$\rho V_{\text{pallo}} = \rho V_{\text{kuutio}}$$

$$V_{\text{pallo}} = V_{\text{kuutio}}$$

$$\frac{4}{3}\pi r^3 = a^3$$

$$a = \sqrt[3]{\frac{4}{3}\pi \cdot r^3} = \sqrt[3]{\frac{4}{3}\pi} \cdot r$$

Pinta-alojen suhde

$$\frac{A_{\text{pallo}}}{A_{\text{kuutio}}} = \frac{4\pi r^2}{6a^2} = \frac{4\pi r^2}{6 \cdot \left(\sqrt[3]{\frac{4}{3}\pi}\right)^2 \cdot r^2}$$

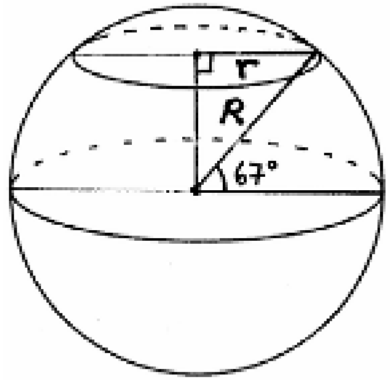
$$= \frac{4\pi}{6\left(\sqrt[3]{\frac{4}{3}\pi}\right)^2} = 0{,}805... \approx 0{,}81$$

Vastaus 0,81

1108

Kun saavutaan Rovaniemelle, on kuljettu 67. leveyspiiri ympäri.
Lasketaan 67. leveyspiirin pituus.
Maan säde on $R = 6400$ km.

Leveyspiirin säde:



$$\cos 67^\circ = \frac{r}{R}$$

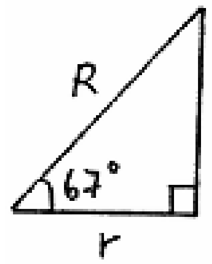
$$r = R\cos 67^\circ$$

$$r = 6400 \text{ km} \cdot \cos 67^\circ$$

$$r = 2\ 500{,}67... \text{ km}$$

Leveyspiirin pituus:

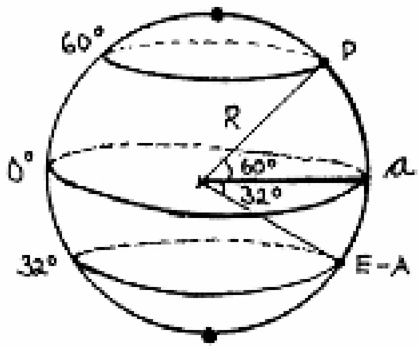
$$2\pi r = 2\pi \cdot 2\ 500{,}23... \text{ km} \approx 16\ 000 \text{ km}$$



Vastaus 16 000 km

1109

a) Kuvassa maapallon oikea reuna on pituuspiiri 30° itäistä pituutta.



Sektorin kaaren pituus a on

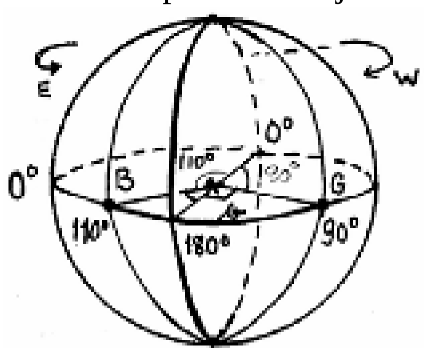
$$a = \frac{\alpha}{360^\circ} \cdot 2\pi R \qquad \left| \begin{array}{l} \alpha = 60^\circ + 32^\circ = 92^\circ \\ r = 6\ 370 \text{ km} \end{array} \right.$$

$$= \frac{92^\circ}{360^\circ} \cdot 2\pi \cdot 6\ 370$$

$$= 10\ 228{,}32...$$

$$\approx 10\ 200 \text{ (km)}$$

b) Molemmat kohteet ovat päiväntasaajalla.



$$b = \frac{\alpha}{360^\circ} \cdot 2\pi R \qquad \left| \begin{array}{l} \alpha = 360^\circ - 110^\circ - 90^\circ \\ \quad = 160^\circ \ (\le 180^\circ) \end{array} \right.$$

$$= \frac{160^\circ}{360^\circ} \cdot 2\pi \cdot 6\ 370$$

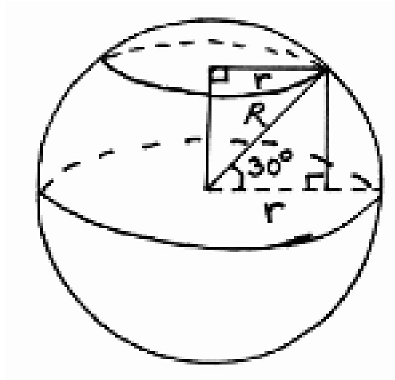
$$= 17\ 788{,}39...$$

$$\approx 17\ 800 \text{ (km)}$$

c) Lasketaan leveyspiirin 30° säde r.

$$\cos 30^\circ = \frac{r}{R}$$

$$r = R\cos 30^\circ = \frac{\sqrt{3}}{2}R$$



Kairon ja New Orleansin välinen kulma α on

$$\alpha = 31^\circ + 90^\circ = 121^{\circ\circ} (\le 180^\circ)$$

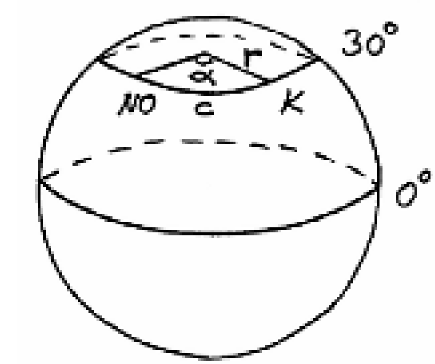
Kaaren pituus on

$$c = \frac{\alpha}{360^\circ} \cdot 2\pi r$$

$$= \frac{121^\circ}{360^\circ} \cdot 2\pi \frac{\sqrt{3}}{2} \cdot 6\ 370$$

$$= 11\ 650{,}18...$$

$$\approx 11\ 700 \text{ (km)}$$



d) Kahden pisteen välinen lyhin reitti pallon pinnalla on pisteiden kautta kulkevalla isoympyrällä.
Kairo $(30^\circ\text{N};\ 31^\circ\text{E})$ ja New Orleans $(30^\circ\text{N};\ 90^\circ\text{W})$ ovat samalla pohjoisella leveyspiirillä, jonka säde on

$$r = R\cos 30^\circ \qquad (\text{c-kohta})$$

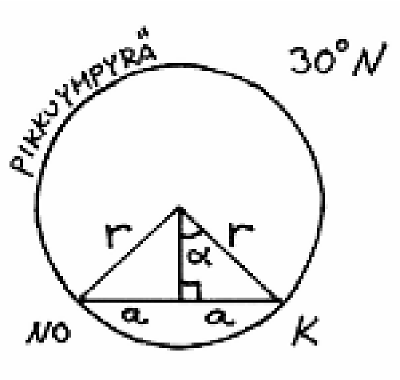
$$\alpha = \frac{90^\circ + 31^\circ}{2} = 60{,}5^\circ$$

$$\sin 60{,}5^\circ = \frac{a}{r}$$

$$a = r\sin 60{,}5^\circ$$

$$a = R\cos 30^\circ \sin 60{,}5^\circ$$

$$a = 4\,801{,}388...\ (\text{km})$$



$$\sin\beta = \frac{a}{R}$$

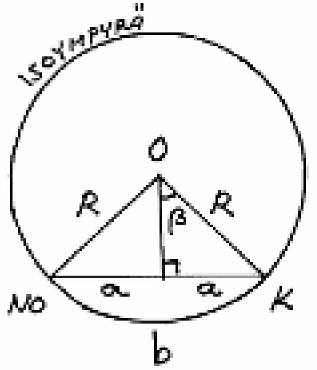
$$\sin\beta = \frac{R\cos 30^\circ \sin 60{,}5^\circ}{R}$$

$$\sin\beta = 0{,}75575...$$

$$\beta = 48{,}916...^\circ$$

$$b = \frac{2\beta}{360^\circ}\cdot 2\pi R = 10\,876{,}77...$$

$$\approx 10\,900\ (\text{km})$$



Vastaus a) 10 200 km b) 17 800 km
c) 11 700 km d) 10 900 km

1110

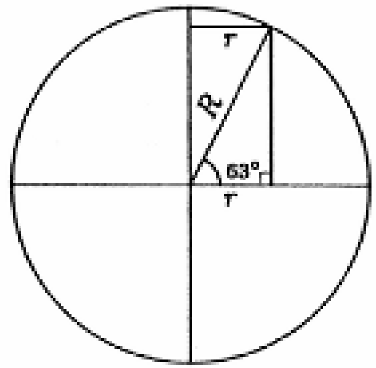
Leveyspiirin säde:

$$\cos 63^\circ = \frac{r}{R}$$

$$r = R\cos 63^\circ$$

$$r = 6\,360\cos 63^\circ$$

$$r = 2\,887{,}37...\ (\text{km})$$



Leveyspiirin pituus on

$$2\pi r = 2\pi R\cos 63^\circ = 2\pi\cdot 6\,360\cos 63^\circ = 18\,141{,}94...\ (\text{km})$$

Laihian ja Kaavin välinen etäisyys on $LK = 330$ km.
Leveyspiiri vastaa aikaa 24 h.

Tapa 1

Merkitään aikaeroa kirjaimella *x*.
Leveyspiirin pituus on

$$2\pi r = 2\pi\cdot 6\,370\cos 63^\circ\ (\text{km}),$$

jonka Maa pyörähtää 24 tunnissa.

Saadaan verranto

$$\frac{x}{24}=\frac{330\text{ km}}{2\pi\cdot 6\,360\cos 63^\circ\text{ km}}$$

$$x=\frac{24\cdot 330}{2\pi\cdot 6\,360\cos 63^\circ}=0,4365...\ (\text{h})$$

$$x=0,4365...\text{ h}=26{,}19...\text{ min}\approx 26\text{ min }12\text{ s}$$

Tapa 2
Matkojen suhde on

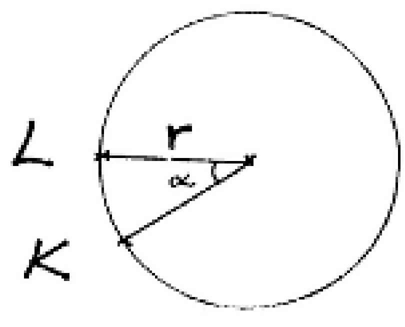
$$k=\frac{330}{2\pi r}=\frac{330}{2\pi R\cos 63^\circ}=\frac{330}{2\pi\cdot 6\,360\cos 63^\circ}$$

Aikaero Laihialta Kaaville mentäessä on

$$d=k\cdot 24\text{ h}=\frac{330}{2\pi\cdot 6\,360\cos 63^\circ}\cdot 24\text{ h}$$

$$=0,43665...\text{h}=26,2...\text{min}\approx 26\text{ min }12\text{ s}$$

Tapa 3



$$r=R\cos 63^\circ=6\,360\cos 63^\circ$$
$$p=2\pi\cdot 6\,360\cos 63^\circ$$

Lasketaan kaarta *LK* vastaava keskuskulma α yhtälöstä

$$\frac{\alpha}{360^\circ}\cdot 2\pi r=330$$

$$\alpha=\frac{330\cdot 360^\circ}{2\pi r}$$

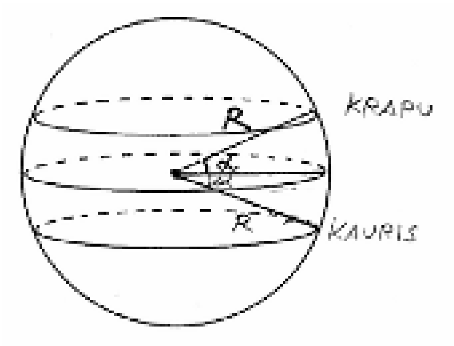
$$\alpha=\frac{330\cdot 360^\circ}{2\pi\cdot 6\,360\cos 63^\circ}=6,548...^\circ$$

Aikaero on $\frac{\alpha}{360^\circ}\cdot 24\cdot 60\text{ min}=26{,}19...\text{ min}\approx 26\text{ min }12\text{ s}$

Vastaus Aikaero Kaavin ja Laihian välillä on 26 min 12 s.

1111

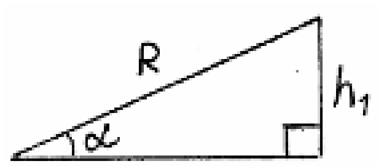
Vyöhykkeen ala on $2\pi rh$.
Maapallon säde $R = 6\ 400\ (\text{km})$.
$\alpha = 23{,}5^\circ$



Lasketaan päiväntasaajan ja Kravun kääntöpiirin väliin jäävän vyöhykkeen korkeus h_1.

$$\sin\alpha = \frac{h_1}{R}$$

$$h_1 = R\sin\alpha$$



Koko kuuman vyöhykkeen korkeus on

$$h = 2h_1 = 2R\sin\alpha$$

ja pinta-ala

$$A = 2\pi Rh = 2\pi R \cdot 2R\sin\alpha = 4\pi R^2 \sin\alpha$$

Kuuman vyöhykkeen ala koko maapallon alasta on

$$\frac{A}{4\pi R^2}\cdot 100\ \% = \frac{4\pi R^2\sin\alpha}{4\pi R^2}\cdot 100\ \%$$

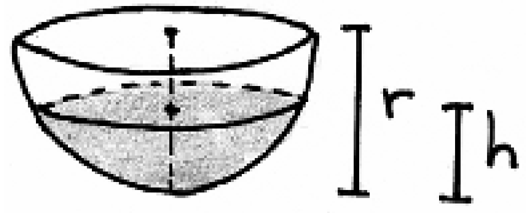
$$= \sin\alpha \cdot 100\ \% = 39{,}87...\% \approx 40\ \%$$

Vastaus 40%

1112

$$V_{\text{liuos}} = 30\ \text{ml} = 0{,}030\ l = 0{,}030\ \text{dm}^3 = 30\ \text{cm}^3$$

puolipallon säde *r* (cm)
liuoksen korkeus *h* (cm)



Pallosegmentin tilavuus

$$V_{\text{pallosegm}} = \pi h^2\left(r - \frac{h}{3}\right)$$

Saadaan yhtälö

$$\pi h^2\left(r - \frac{h}{3}\right) = 30 \qquad \Big| h = \frac{r}{2}$$

$$\pi\left(\frac{r}{2}\right)^2\left(r - \frac{1}{3}\cdot\frac{r}{2}\right) = 30$$

$$\pi\frac{r^2}{4}\cdot\frac{5}{6}r = 30$$

$$\frac{5\pi r^3}{24} = 30 \qquad |\cdot 24 \qquad |:5\pi$$

$$r^3 = \frac{30\cdot 24}{5\pi}$$

$$r^3 = \frac{144}{\pi}$$

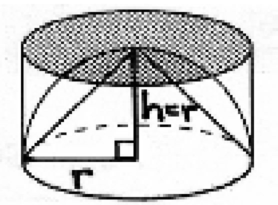
$$r = \sqrt[3]{\frac{144}{\pi}}\ \ (\text{cm})$$

Korkin tilavuus on

$$V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3 = \frac{2}{3}\pi \cdot \frac{144}{\pi} = 96 \left(\text{cm}^3 = \text{ml}\right)$$

Vastaus 96 ml

1113



$$V_{\text{kartio}} = \frac{1}{3}A_p h = \frac{1}{3} \cdot \pi r^2 \cdot r = \frac{\pi r^3}{3}$$

$$V_{\text{puolipallo}} = \frac{1}{2}V_{\text{pallo}} = \frac{1}{2} \cdot \frac{4}{3}\pi r^3 = \frac{2\pi r^3}{3}$$

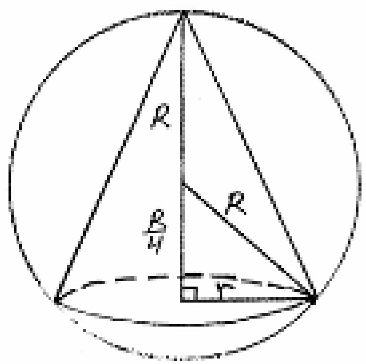
$$V_{\text{lieriö}} = A_p h = \pi r^2 \cdot r = \pi r^3$$

$$\begin{aligned} &V_{\text{kartio}} : V_{\text{puolipallo}} : V_{\text{lieriö}} \\ &= \frac{\pi r^3}{3} : \frac{2\pi r^3}{3} : {}^{3)}\pi r^3 \\ &= \frac{\pi r^3}{3} : \frac{2\pi r^3}{3} : \frac{3\pi r^3}{3} \\ &= \pi r^3 : 2\pi r^3 : 3\pi r^3 \\ &= 1 : 2 : 3 \end{aligned}$$

□

1114

$$\begin{aligned} r^2 + \left(\frac{R}{4}\right)^2 &= R^2 \\ r^2 + \frac{R^2}{16} &= R^2 \\ r^2 &= R^2 - \frac{R^2}{16} \\ r^2 &= \frac{15R^2}{16} \\ r &= \frac{R\sqrt{15}}{4} \end{aligned}$$



Kartion tilavuus

$$V_{\text{kartio}} = \frac{1}{3}A_p h = \frac{1}{3}\pi r^2 \cdot \frac{5}{4}R = \frac{\pi}{3} \cdot \frac{15R^2}{16} \cdot \frac{5R}{4} = \frac{25\pi R^3}{64}$$

Pallon tilavuus

$$V_{\text{pallo}} = \frac{4\pi R^3}{3}$$

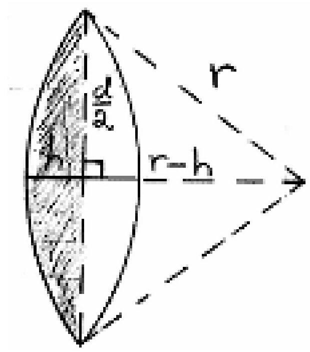
Siis

$$\frac{V_{\text{kartio}}}{\mathrm{V}_{\text{pallo}}}\cdot 100\ \% = \frac{25\pi R^3}{64} : \frac{4\pi R^3}{3}\cdot 100\ \%$$
$$= \frac{25\pi R^3}{64}\cdot\frac{3}{4\pi R^3}\cdot 100\ \%$$
$$= \frac{1875}{64}\ \% = 29\frac{19}{64}\ \% \approx 29{,}3\ \%$$

Vastaus $\frac{1875}{64}\% = 29\frac{19}{64}\% \approx 29{,}3\%$

1115

Määritetään ensin linssin puolikkaan tilavuus:



$$V_{\text{pallosegm}} = \pi h^2\left(r - \frac{h}{3}\right)$$
$$h = \frac{5{,}0\ \text{mm}}{2} = 2{,}5\ \text{mm}$$
$$d = 4{,}1\ \text{cm} = 41\ \text{mm}$$

Pythagoraan lauseen mukaan

$$r^2 = (r-h)^2 + \left(\frac{d}{2}\right)^2$$
$$r^2 = r^2 - 2rh + h^2 + \frac{d^2}{4}$$
$$2rh = h^2 + \frac{d^2}{4}$$
$$r = \frac{h^2 + \frac{d^2}{4}}{2h} = \frac{(2{,}5\ \text{mm})^2 + \frac{(41\ \text{mm})^2}{4}}{2\cdot 25\ \text{mm}} = 85{,}3\ \text{mm}$$
$$\mathrm{V}_{\text{pallosegm}} = \pi\cdot(2{,}5\ \text{mm})^2\left(85{,}3\ \text{mm} - \frac{2{,}5\ \text{mm}}{3}\right) = 1\,658{,}49...\ \text{mm}^3$$
$$\mathrm{V}_{\text{linssi}} = 2\cdot\mathrm{V}_{\text{pallosegm}} = 3\,316{,}99...\ \text{mm}^3 \approx 3{,}3\ \text{cm}^3$$

Vastaus $3{,}3\ \text{cm}^3$